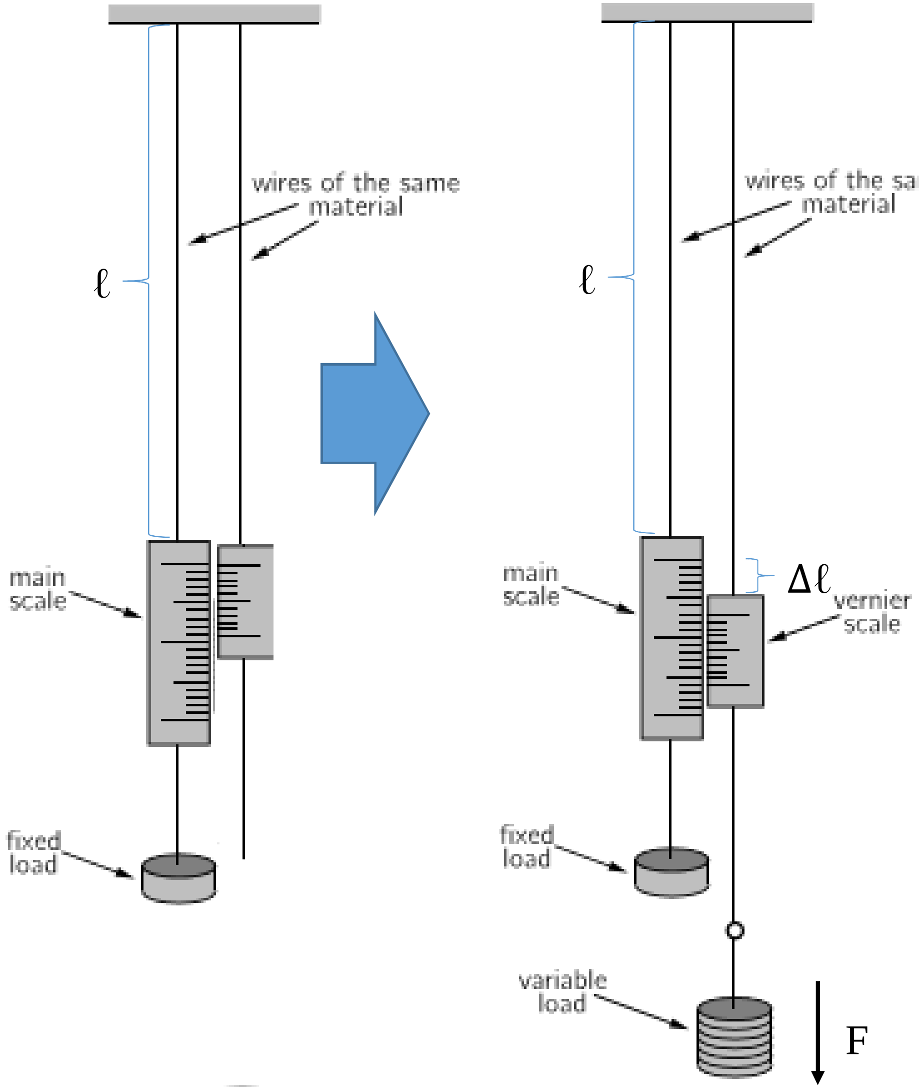
Measurements of Young's modulus



$$Y = \frac{F/A}{\Delta\ell/\ell}$$

$$\Delta\ell/\ell \times Y = \frac{F/A}{\Delta\ell/\ell} \times \Delta\ell/\ell$$

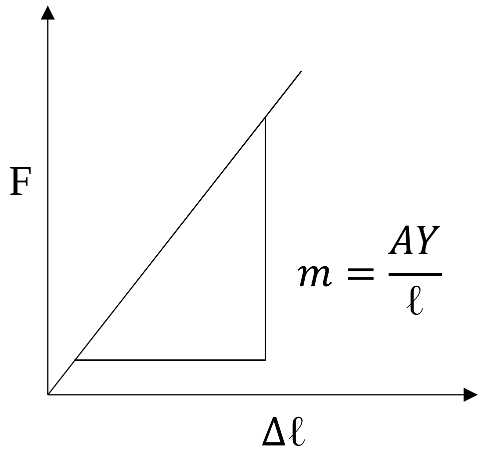
$$\frac{\Delta\ell Y}{\ell} = \frac{F}{A}$$

$$A \times \frac{\Delta\ell Y}{\ell} = \frac{F}{A} \times A$$

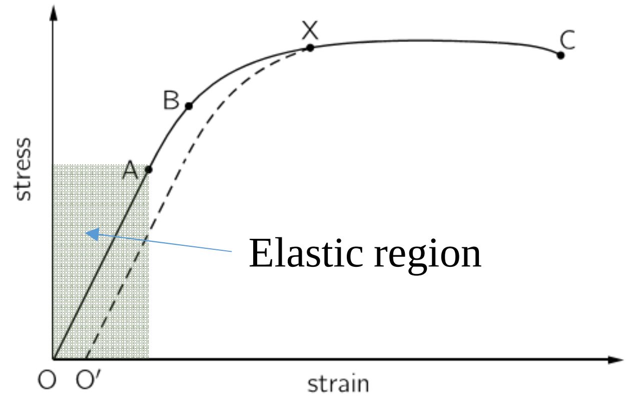
$$F = \frac{A\Delta\ell Y}{\ell}$$

$$F = \left(\frac{AY}{\ell}\right)\Delta\ell$$

$$y \;=\; m\,x$$



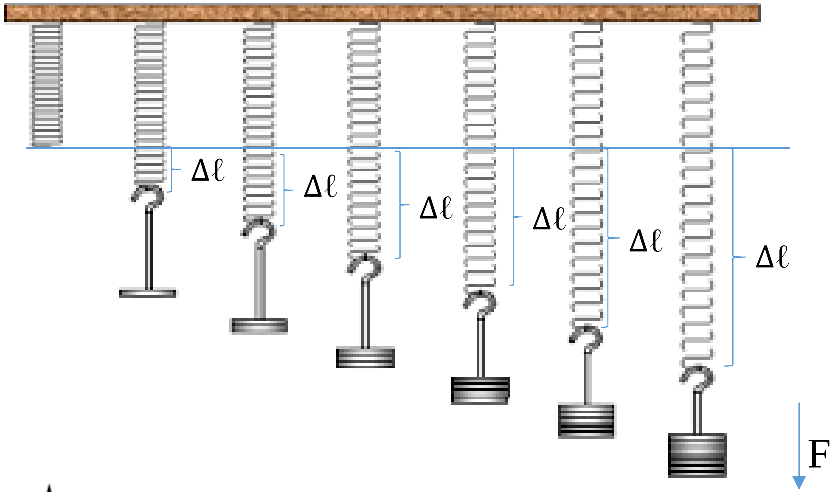
$$Y = \frac{m\ell}{A}$$

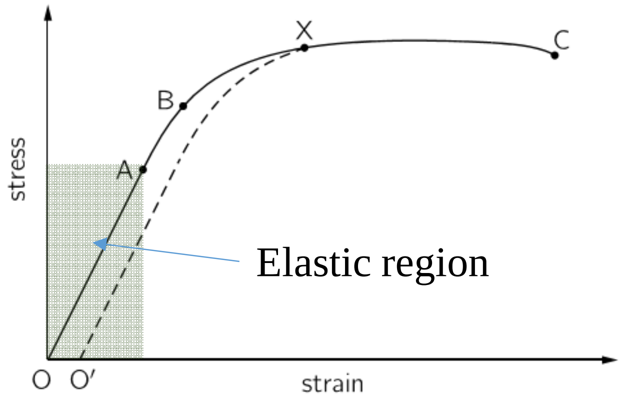


Is Hooke's law applicable in the stretching of wire????

Hooke's law

Consider a spring suspended from the ceiling. If the weights are progressively added, an extension/deformation of the spring progressively increases as shown in the figure below





$$F_{Applied} \propto \Delta\ell$$

We can eliminate the proportionality sign and introduce an equal sign and the constant k, called a **spring constant**.

$$F_{Applied} = k\Delta\ell$$

This is Hooke's law: the stress/force imposed on a solid is directly proportional to the strain/deformation produced, within the elastic limit

According to Newton's third law the spring exerts a force equal and opposite to the force applied on it and it is called a **restoring force**

$$F_{Restoring} = -k\Delta\ell$$

which is the same as

$$F = \left(\frac{AY}{\ell}\right)\Delta\ell$$

where $k = \frac{AY}{\ell}$. So Hooke's law is applicable in a stretching of wire.

Coiled springs and rubber bands obey Hooke's law so long as the extension $\Delta\ell$ is relatively small and stays within the linear portion of the elastic region, so does the stretched wire.

Examples

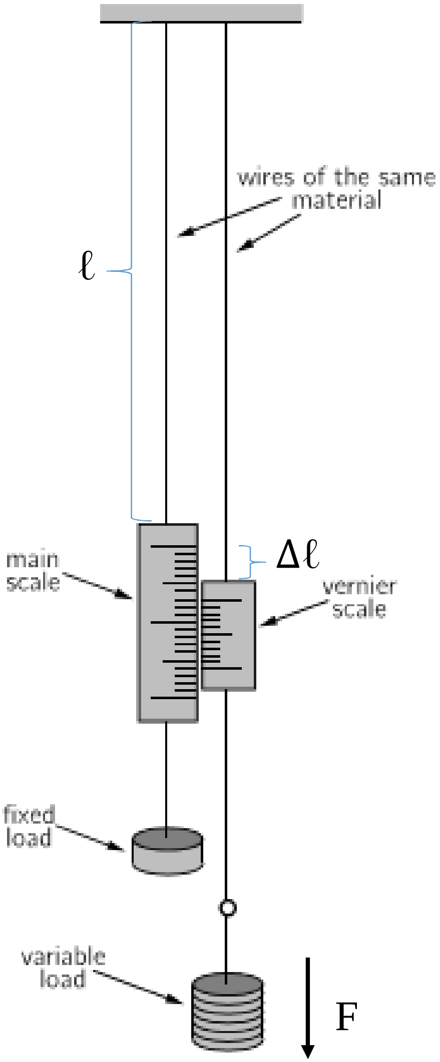
Example 5.3: Compressing and stretching a spring.

A spring with a spring constant $k = 200\ Nm^{-1}$ has a length of 8.0 cm when not under load.

(a) What force must be applied to compress the spring to half its length?

(b) What force must be applied to stretch the spring to twice its length?

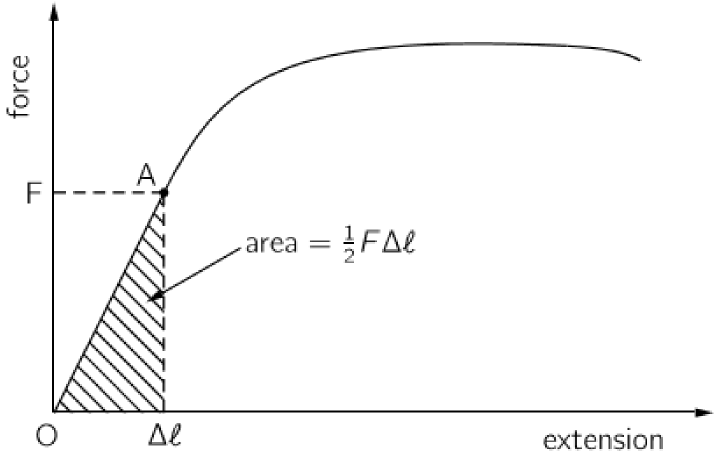
Work done in stretching a wire



If the wire is stretched, work is done on it, at the same time energy is supplied into it and stored as strain energy. Suppose a wire is gradually stretched by a force, causing a final extension $\Delta\ell$ when the force equals F (initially there is no force and hence no extension and we assume the stretching is elastic). The average applied force during the stretching is therefore $\frac{1}{2}F$. We can now find the strain energy from the work done on the wire:

$$SE = W = force \times distance = \frac{1}{2}F\Delta\ell.$$

A graph of the applied force versus the extension is as shown in the graph and has the same shape as the stress-strain graph. This figure shows that the strain energy equals the area under the straight line portion of the graph (the shaded region)



Is the strain energy related to stress and strain????

Work done in stretching a wire

The strain energy where the elastic deformation is linear can be related to the stress and the strain using the definitions of stress and strain

$$\text{SE} = \frac{1}{2} F\Delta\ell$$

$$\text{SE} = \frac{1}{2}(\text{stress} \times \text{area}) \times (\text{strain} \times \text{length})$$

$$\text{SE} = \frac{1}{2}\,\text{stress} \times \text{strain} \times \text{volume}$$

$$\text{SE/volume} = \frac{1}{2}\,\text{stress} \times \text{strain}$$