

Average Acceleration

The acceleration of an object is a measure of how fast the velocity of an object is changing.

The average acceleration of an object in a given direction is the change in velocity divided by the total time taken.

Mathematically, it is given by:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

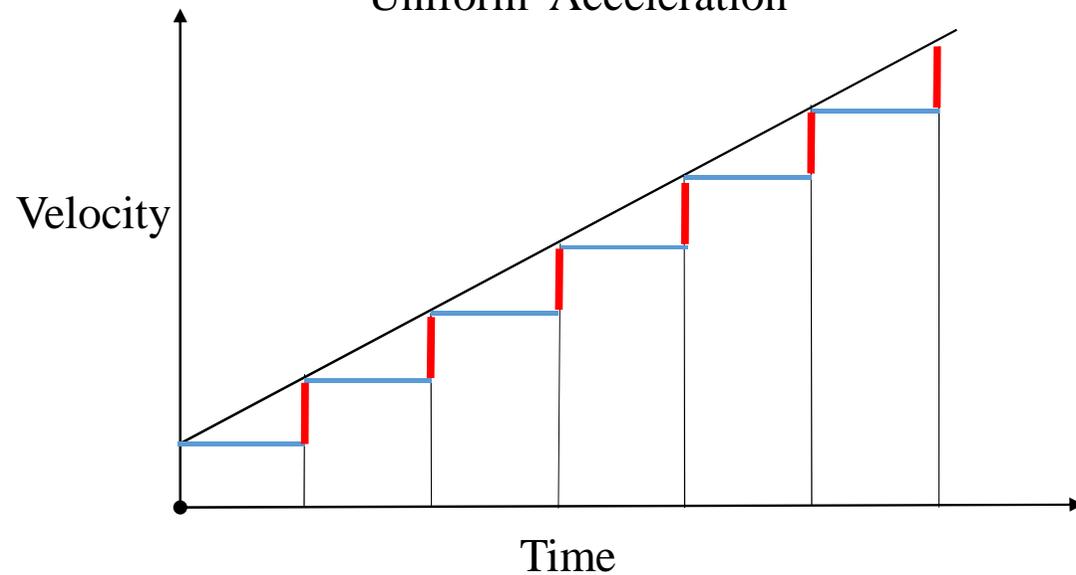
Instantaneous Acceleration

The instantaneous acceleration is the acceleration at any given time during the motion of an object. It may also be defined as an acceleration over a very small time interval. In this course we will refer to instantaneous acceleration as just acceleration.

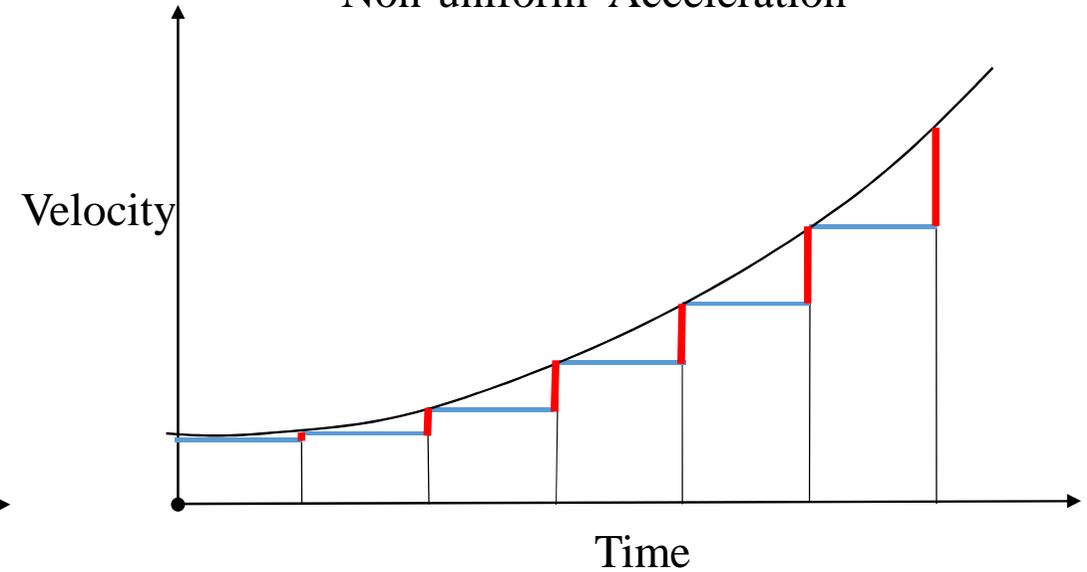
Uniform and non-acceleration

An object whose velocity changes by an equal amount over equal time intervals is said to have a uniform acceleration.

Uniform Acceleration



Non-uniform Acceleration



The **gradient** of the velocity-time graph gives the **acceleration**.

The **area under** the velocity-time graph gives the **displacement**.

Uniform acceleration cont.

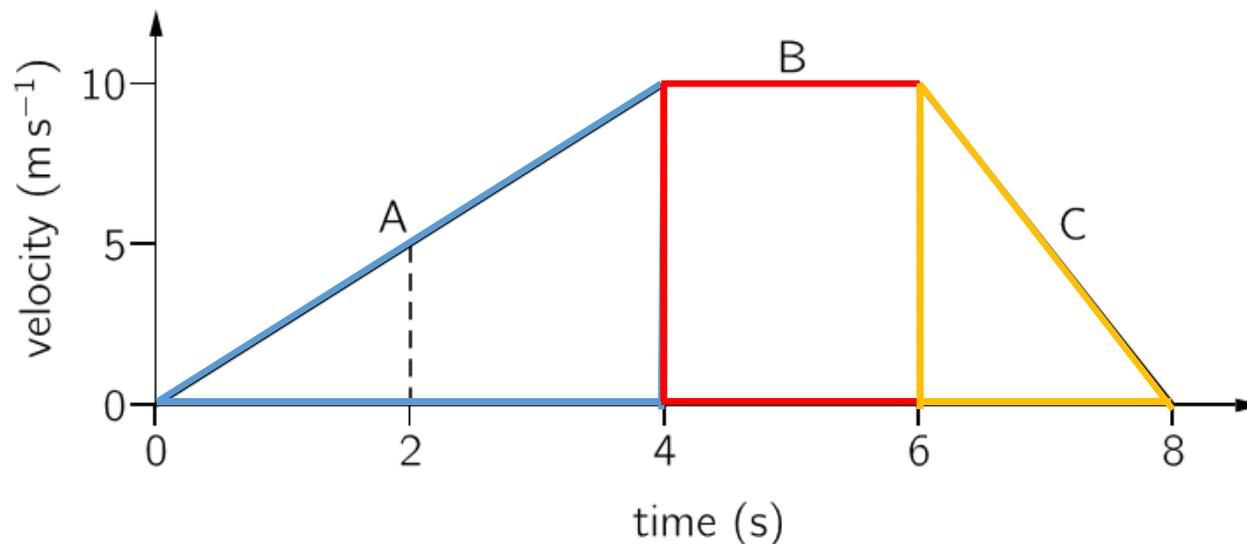
Example 2.3: Average acceleration

An aeroplane coming in to land is travelling at a speed of 100 ms^{-1} . What is the average acceleration of the aeroplane if it comes to rest in a time of 10 s ?

Example 2.4: Velocity-time graph

A car accelerates uniformly from rest, drives at constant velocity for a short while before decelerating, and coming to rest at a traffic light.

- Find the acceleration in each of the segments A, B and C indicated on the graph below.
- Find also the total displacement during segments A, B and C.



Kinematic equations for uniform acceleration in one dimension

Consider an object undergoing a one dimensional motion, either horizontally or vertically. The coordinate system is taken as our reference frame and the origin as our reference point. The properties of the object's motion, such as displacement, velocity and acceleration can be described using **kinematic equations of motion**. We are going to consider an object travelling at **constant acceleration from the origin of the coordinate system along the x-axis**. If an object starts from rest, x_i, t_i and v_i are equal to zero. To simplify our notation, we are going to make the following substitutions:

Elapsed time $\Delta t = t$

Displacement $\Delta x = x_f - x_i = s$

Initial velocity $v_i = u$

Final velocity $v_f = v$

For **constant acceleration average acceleration is the same as instantaneous acceleration** $\bar{a} = a$.

This is summarised in the table below

Variables	Symbols	Unit
Time	t	s
Displacement	s	m
Initial velocity	u	ms^{-1}
Final velocity	v	ms^{-1}
Acceleration	a	ms^{-2}

Kinematic equations for uniform acceleration in one dimension

Kinematic equation of motion are derived on page 15 of your lecture notes. You are not expected to know how to derive these equations, but you need to know how to use them to solve problems concerning one dimensional motion. If you are interested in the derivation, please read your notes and consult with me if you do not follow. The equations of motion that will be of use to solve one dimensional problems are:

$$\begin{aligned}v &= u + at \\s &= ut + \frac{1}{2}at^2 \\v^2 &= u^2 + 2as\end{aligned}$$

Method for problem solving

- Choose the system of axis. It is convenient to choose positive direction along the positive x-axis.
- List all the given variables. Read the problem carefully because some information may be contained in the statement. For example, ... the object travels from rest means that $u = 0 \text{ m/s}$.
- Choose an appropriate equation to solve for the desired variable. There should be three given variables and the fourth (designed) one can be computed by choosing an appropriate equation.

Kinematic equations for uniform acceleration in one dimension

Example 2.5: Acceleration of a car from rest

A car accelerates uniformly from rest to 36 km h^{-1} in 4 s. Calculate the magnitude of the acceleration and the distance covered during this 4 s interval.

Example 2.6: Distance covered by an accelerating spacecraft

A spacecraft is travelling with a velocity of 3000 m s^{-1} when it fires its retrorockets and begins to slow down with an acceleration whose magnitude is 10 m s^{-2} . Determine the velocity of the spacecraft when its displacement is 200 km relative to the point at which the retrorockets were fired

Kinematic equations in one dimension- Motion under gravity

The gravitational acceleration (g) is the acceleration due to the gravitational force, on earth, its magnitude is approximately 9.8 ms^{-2} . The gravitational force always points downward, therefore the direction of g is always **downward**. An object moving downward under the gravitational pull has $g = + 9.8 \text{ ms}^{-2}$. However, an object moving upward suffers a retardation and thus has $g = - 9.8 \text{ ms}^{-2}$.

Strategies for problem solving

1. **Draw a diagram** to represent the situation. A simple 'block' drawing using arrows to indicate the directions of the various velocities and or accelerations is often all that is needed to gain a good understanding of a problem.
2. **Choose a set of coordinate axes** and decide which directions are to be called positive and negative. It is often convenient to place the origin at the place where an object starts its motion. In problems involving one dimension, the positive axis is usually chosen to go from left to right. For motion in two directions (see Section 2.6) the vertical direction is usually chosen as the y direction with 'up' being positive. To avoid confusion, **do not change your decision in the middle of a calculation.**
3. Write down the available values for the kinematic variables (s , u , v , a and t). Be careful to assign the appropriate sign depending on the choice of coordinate axes made in 2.
4. At least three of the kinematic variables should have values. Be sure to read the question carefully. There may be implied data like 'an object is accelerated from rest', in which case we may write $u = 0$.
5. Often a problem is divided into parts. For instance, a car may accelerate for a period of time, travel at constant velocity for a distance and then slow down. In such a case, divide the problem into parts, bearing in mind that the initial values for each part are given by the final values of the previous part.

Kinematic equations in one dimension- Motion under gravity

Example 2.7: Motion under gravity

Calculate the time taken for a ball thrown vertically upwards, with an initial speed of 19.6 m s^{-1} , to return to its starting point, neglecting air resistance.