

TIME: 3 hours

MAXIMUM MARKS: 180

Internal Examiner(s): Dr A Sergi Dr V W Couling	External Examiner: Prof D Naidoo (University of the Witwatersrand)
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General instructions:

- ☞ Answer **all** questions.
- ☞ As it is understanding that is being tested, explanation of the steps and physics involved must accompany any mathematics.
- ☞ Candidates are reminded to be as thorough as possible and to write legibly.

Section A: Atomic Spectroscopy

Question 1

Show, in outline, how perturbation treatments of the electronic configuration energy levels for light atoms give rise to spectroscopic terms and levels. (15)

Question 2

- (2a) Determine the spectroscopic terms arising from the electronic configuration $(np)(mp)$ where $n \neq m$. Use Breit's rule to determine the terms arising from the electronic configuration $(np)^2$. Explain why there are fewer terms for the two equivalent electrons than for the two non-equivalent electrons, and illustrate your answer by showing why the 3D term does not arise in $(np)^2$. (10)
- (2b) State Hund's rules for the ordering of spectroscopic terms and levels associated with a given configuration. (3)
- (2c) Fluorine, which is a light atom, has the ground-state electronic configuration given by $(1s)^2(2s)^2(2p)^5$, and the configuration of the first excited state is $(1s)^2(2s)^2(2p)^4(3s)$. Determine the terms and levels arising from these two configurations, and use Hund's rules to arrange the terms and levels in order of increasing energy. By noting the selection rule on S, draw an energy-level diagram for fluorine. Indicate two allowed transitions and three transitions forbidden by different selection rules. State next to the forbidden transitions how a selection rule would be violated. (24)

Question 3

Calculate the Zeeman splittings of the spectral lines resulting from a ${}^1P \rightarrow {}^1S$ transition when the light source is placed in a weak transverse magnetic field. Draw the corresponding energy-level diagram.

You are reminded of the result:

$$g = \left\{ 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \right\} \quad (8)$$

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Section B: Sub-atomic Physics

Question 4

Consider a two-level system in a quantum state (given in the representation of the energy)

$$|\Psi\rangle = a|\phi_1\rangle + b|\phi_2\rangle,$$

where $|\phi_1\rangle$ and $|\phi_2\rangle$ are the eigenstates of the Hamiltonian operator \hat{H} , corresponding to the eigenvalue E_1 and E_2 , respectively.

Given that the probability of finding the energy of the system to be E_1 is $\frac{1}{25}$, find the numerical values for the coefficients a and b . (5)

Question 5

- (a) Write down the equation that represents the possible results of the measurement of the momentum of a particle. (1)
- (b) Represent the equation from part (a) in the position basis. (4)
- (c) Determine the transformation law between the position and momentum basis. (4)

Question 6

Consider a 2-level system described by the Hamiltonian operator \hat{H}_0 . Before time $t_1 = 0$ the system is in its ground state. Between time $t_1 = 0$ and $t_2 = T$ the system interacts with an external apparatus. Such a perturbation is represented by means of the perturbation operator $\hat{H}_I(t)$, which when represented in the basis of the eigenstates of \hat{H}_0 is purely off-diagonal.

Find to the first order in perturbation theory the probability that after time $t_2 = T$, the 2-level system makes a transition to its excited state. (30)

Question 7

Consider the Lorentz transformation laws between inertial frames when the two frames have parallel axes and move along the x direction with relative constant speed u .

- (a) Show what happens to the transformation equations when $u \ll c$, where c is the speed of light. (5)
- (b) Show also that the Newton equation is not affected by the obtained transformation equations. (4)
- (c) Derive the relativistic law of composition of velocities. (9)
- (d) Derive the limit of the relativistic law of composition of velocities when $u \ll c$. Such a limit provides the Galilei's transformation laws. Now assume that a light ray is directed in the positive x direction by the frame of reference which is moving with respect to the other with velocity u , also in the positive x direction. What would be the value of the speed of light which is predicted by the fixed frame of reference by using Galilei's transformation? Would that be physically possible? Give an explanation. (10)

Question 8

Write the definition of the relativistic energy of a free particle and discuss its main physical implications, which are relevant to particle and nuclear physics. (8)

Question 9

- (a) Provide an intuitive derivation of the Klein-Gordon equation. Can you apply this equation to the relativistic electron? Provide a motivation for the answer. (10)
- (b) What happens to the equation determined in part (a) if the mass of the particle is zero? What kind of physical object is described by the equation in this case? (4)

Question 10

Briefly explain the main classifications of particles and forces in the Standard Model of particle physics. In particular, explain what are the characteristics of the particles that are considered matter and what is the quantum interpretation of the forces between them. (10)

Question 11

Write down in mathematical form the laws of lepton conservation for each generation. (8)

Question 12

Consider the β -decay of a single neutron.

- (a) Write the particle reaction associated with this process. (2)
- (b) What happens to the electric charge in such a process? (2)
- (c) What happens to the lepton number? (2)
- (d) What happens to the baryon number? (2)

[120]
[180]