Resistance And Resistivity

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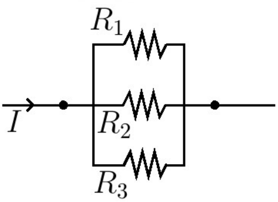
Resistors In Series And Parallel

For resistors in series, the current through each must be the same, while the potential difference differs.

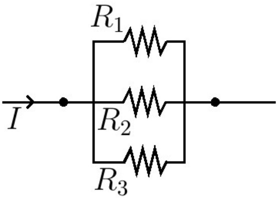
$$R_{\text{eq}} = R_1 + R_2 + R_3 + \dots$$

For parallel resistors, it is the current that differs through each, while the potential difference remains the same for each branch.



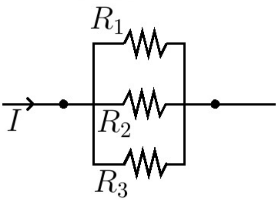
$$I = I_1 + I_2 + I_3 \quad \therefore \quad \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Resistors In Parallel



How much current flows through each resistor?
Since resistance is a measure of the opposition to charge flow, a large value for R_1 results in less current flowing through that branch, and subsequently more current flowing in each of R_2 and R_3.

Resistors In Parallel



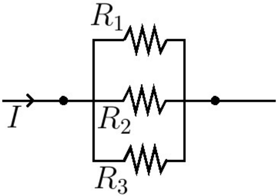
The total current entering I the three branches is given by

$$I = \frac{\Delta V}{R_{\text{eq}}}$$

Similarly, the current I_1 through resistor R_1 is given by the potential difference across R_1 and its resistance:

$$I_1 = \frac{\Delta V}{R_1} = \frac{1}{R_1}\left(I R_{\text{eq}}\right)$$

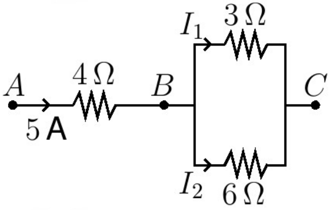
Resistors In Parallel



The currents in the other two branches are

$$I_2 = \frac{1}{R_2}\left(IR_{\text{eq}}\right), \quad I_3 = \frac{1}{R_3}\left(IR_{\text{eq}}\right)$$

An Example

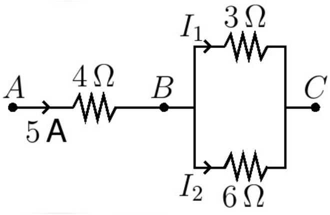


(a) Find the equivalent resistance between the points A and C.

(b) Find the potential difference across points A and C and each resistor.

(c) Calculate the currents I_1 and I_2.

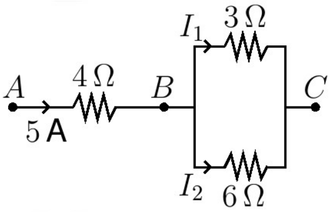
An Example



(a) Find the equivalent resistance between the points A and C.
The $3\,\Omega$ and $6\,\Omega$ resistors are in parallel with each other, and that combination in series with the $4\,\Omega$ resistor.
The equivalent resistance $R_{\text{eq}}^{(BC)}$ between B and C is

$$R_{\text{eq}}^{(BC)} = \frac{3 \times 6}{3 + 6} = 2\,\Omega$$

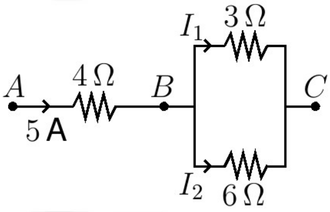
An Example



(a) Find the equivalent resistance between the points A and C.
The equivalent resistance $R_{\text{eq}}^{(AC)}$ between A and C is

$$R_{\text{eq}}^{(AC)} = 4 + R_{\text{eq}}^{(BC)} = 4 + 2 = 6\,\Omega$$

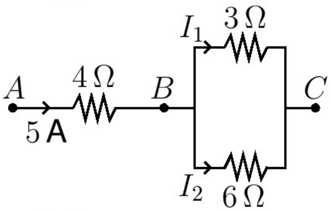
An Example



(b) Find the potential difference across points A and C and each resistor.
Given a total current of 5 A and an equivalent resistance of $6\,\Omega$, the potential difference across points A and C is

$$\Delta V_{AC} = IR_{\text{eq}}^{(AC)} = 5 \times 6 = 30\,\text{V}$$

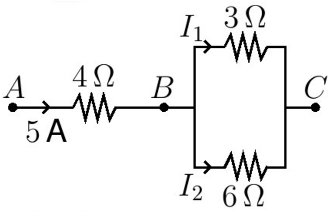
An Example



(b) Find the potential difference across points A and C and each resistor.
How much current flows through the $4\,\Omega$ resistor?
The potential difference across this resistor is

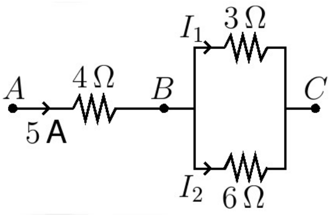
$$\Delta V_{(4)} = IR_{(4)} = 5 \times 4 = 20\,\text{V}$$

An Example



(b) Find the potential difference across points A and C and each resistor.
The potential difference across the $3\,\Omega$ resistor has to be the same as that across the $6\,\Omega$ resistor.
Since there is a potential drop of 20 V across the $4\,\Omega$ resistor, the potential difference across each branch has to be 10 V.

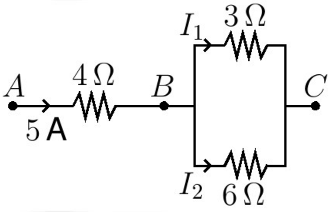
An Example



(c) Calculate the currents I_1 and I_2.
The total current I is 5 A. How much of that current should pass through the $3\,\Omega$ resistor?

$$I_1 = \frac{\Delta V_{(3)}}{R_{(3)}} = \frac{10}{3}\ \text{A}$$

An Example



(c) Calculate the currents I_1 and I_2.
The current through the $6\,\Omega$ resistor should be half of I_1.

$$I_2 = \frac{\Delta V_{(6)}}{R_{(6)}} = \frac{10}{6}\,\text{A}$$

The total current is $10/3 + 10/6 = 5\,$A, as expected.

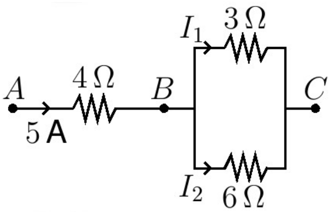
Current-Splitting Rule For Two Resistors

For two resistors in parallel, the current-splitting rule can be used to find the current in each branch.

$$I_1 = \left(\frac{R_2}{R_1 + R_2} \right) I$$

where I_1 is the current through R_1 (and I_2 is the current through R_2) and I is the total current.

Current-Splitting Rule For Two Resistors



For the previous example, given a total current of 5 A, the current-splitting rule can be used to find the current through the $3\,\Omega$ resistor and the $6\,\Omega$ resistor.

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I = \left(\frac{6}{3+6}\right) \times 5 = \frac{30}{9} = \frac{10}{3}\text{ A}$$

Resistance And Physical Properties Of A Resistor

Charge carriers lose kinetic energy as a result of collisions with atoms in a conductor.

The effect of the collisions is like an internal frictional force acting on the charge carriers.

Resistance, therefore, increases with length, since there are more fixed atoms for charge carriers to collide with in a longer conductor.

Resistance also increases with a smaller cross-sectional area.

The resistance of a conductor is then proportional to its physical length l and its cross-sectional area A.

$$R \propto \frac{l}{A}$$

Resistivity

The resistivity ρ of a conductor (resistor) is the proportionality constant:

$$R = \rho \frac{l}{A}$$

The SI unit of resistivity is $\Omega\,\mathrm{m}$.
The resistivity ρ is a property specific to each material. Metals have very low resistivities.
The resistivity of a material depends on its electronic configuration and on temperature.

Temperature Dependance

Since the resistivity ρ depends factors including temperature, the resistance of a material also depends on temperature.

Atoms at higher temperatures increases the charge carrier scattering as they move through the material. This results in an increase in the resistivity.

The resistivity generally increases with temperature as (over a limited temperature range):

$$\rho = \rho_0 \left[1 + \alpha \Delta T\right]$$

Here, ρ is the resistance at some temperature T, ρ_0 is the resistivity at a reference temperature T_0, $\Delta T = T - T_0$ and α is temperature coefficient of resistivity.

Temperature Dependance

Assuming a uniform cross-section A for a resistor, the resistance variation with temperature is:

$$R = R_0 \left[1 + \alpha \Delta T\right]$$

What about the effect of thermal expansion on resistance?