SCHOOL OF CHEMISTRY & PHYSICS

JUNE 2014 FINAL EXAMINATION

PHYS361: QUANTUM MECHANICS AND CLASSICAL MECHANICS

TIME: 3 hours	**MAXIMUM MARKS:** 180

Internal Examiner(s):	**External Examiner:**
Dr Sergi	Dr D Naidoo (UNIVERSITY OF THE WITWATERSRAND)

General instructions:

- Answer **all** questions.
- As it is understanding that is being tested, explanation of the steps and physics involved must accompany any mathematics.
- Candidates are reminded to be as thorough as possible and to write legibly.

Section A: QUANTUM MECHANICS

1. Consider a quantum particle of mass m under the action of a potential that does not depend on time: $V = V(\mathbf{r})$. Show in details, explaining in words the relevant steps, how the time dependent Schrödinger equation can undergo a separation of variables. (16)

2. Consider a quantum particle of mass m, moving on a line under the action of a constant potential $V = V_0$. Prove that $\Phi(x) = A\cos(kx) + B\sin(kx)$ is a stationary solution of the motion (find also the expression of k). (10)

3. Show in detail how the phenomenon of quantum tunneling through a potential barrier can be studied mathematically. (15)

4. Consider a quantum particle moving on a line under the action of the following potential

$$V(x) = \begin{cases} 0 & \text{if } -\frac{a}{2} < x < \frac{a}{2} \\ \infty & \text{if } -\frac{a}{2} > x > \frac{a}{2} \end{cases} . \qquad (*)$$

The quantum motion in potential in Eq. (*) can be described in terms of a combination of functions of even and odd parity.

   (a) Show explicitly how the boundary condition for the even functions implies the quantization of the energy. (8)

   (b) Calculate the normalisation constant in front of the even eigenfunctions. (10)

5. Consider the case of a particle moving on a line which collides, coming from $-\infty$, with a potential step. Find the expression of the reflection coefficient when the energy of the incident particle is higher than that of the potential step. (20)

6. Consider a particle moving on a line under the action of an harmonic potential.

   (a) Introduce the ladder operators, $\hat{A}$ and $\hat{A}^\dagger$ and show in mathematical detail how the Hamiltonian can be expressed through them (show two ways in which this can be done). (15)

   (b) Write down the associated Schrödinger equation in algebraic form (at least in two ways). (2)

(c) Derive the commutation relation $[\hat{A}, \hat{A}^\dagger]$. (4)

(d) Assuming that Φ_n is an eigenstate of the Schrödinger equation of the system, derive the action of the operator $\hat{A}$ onto such eigenstate. (10)

(e) Similarly, derive the action of the operator $\hat{A}^\dagger$ onto such eigenstate. (10)

(Total Marks [120])

Section B: Classical mechanics

1. Consider a system described by generalized coodinates q_i, $i = 1, ..., N$. Show how to derive the Euler-Lagrange equation from the Principle of Least Action. (20)

2. Show how to go from a description in terms of the Lagrangian to a Hamiltonian theory and derive the Hamiltonian equations of motion from the Principle of Least action. (20)

3. Show how to obtain the Euler-Lagrange equations from the variational principle written explicitly in terms of the following Lagrangian density

$$\mathcal{L} = \frac{1}{2}\rho \left[\frac{\partial\phi(x,t)}{\partial t}\right]^2 - \frac{1}{2}T\left[\frac{\partial\phi(x,t)}{\partial x}\right]^2 ,$$

where ρ and T are constants. (20)

(Total Marks: [60])