Work and energy

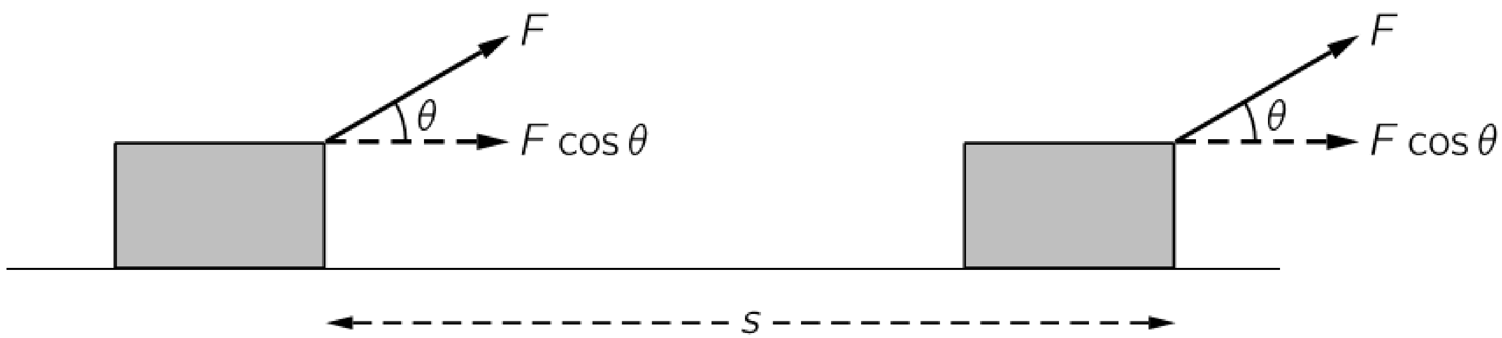
In the last sections we have used Newton's laws and the concepts of mass and force to study the object which are in motion. In this section we are going to introduce two new concepts, that is, work and energy, and subsequently used them to analyse the motion of objects. Work and energy are a concept we used in everyday life and they are related such that they are sometimes referred to as the two sides of the same coin.

Work is defined as what is accomplished by an action of a force when it makes an object moves through a distance. The **energy** on the other hand, is the measure or the capacity of the object or the system to do the work.

There are different forms of energy which includes
Kinetic energy
Potential energy
Electrical energy
Chemical energy
Heat energy
Etc.

However, in this course we are going to concentrate on the kinetic and potential energy. To begin the analysis of moving object let us consider the work done on an object, due to an applied force.

The work done by a force



Consider a constant force of magnitude *F* acting on an object of mass *m* at an angle θ as shown in the figure below. The force moves the object over a distance *s* along the horizontal. The work done on the object is defined as a product of the displacement and the component of the force in the direction of the displacement. Mathematically, it is given as

$$W = Fs\cos\theta$$

Work is a scalar quantity and the units are joules (J) where 1 J = 1 Nm

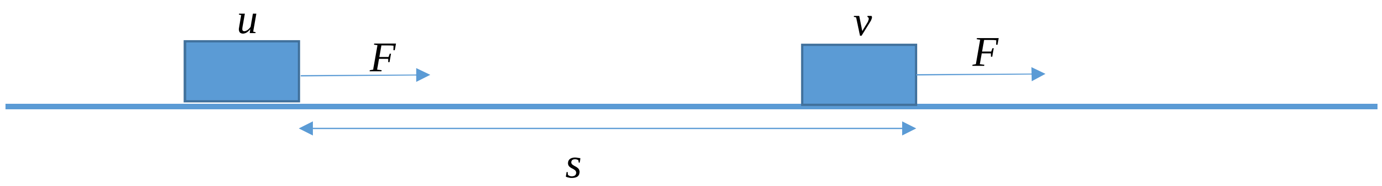
If the applied force is in the direction of the displacement, then $\theta = 0$ and $W = Fs$.
One joule is the work done by a force of one newton when it moves its point of application through a distance of one metre in the direction of the force.

Example

Example 4.1: Work done on an object dragged over a distance

Find the work done when a trunk is dragged a distance of 10 m by a force of 50 N applied at an angle of 45^{o} above the surface over which the trunk is moved.

Kinetic energy and work-energy theorem



Consider an object moving over a displacement s under the influence of a constant net force F. The object's acceleration will thus be constant and can be given by Newton second law $F = ma$. If the initial velocity and the final velocities are u and v, the displacement can be obtained from $v^2 = u^2 + 2as$. Work done on the object is given by

$$W = Fs$$

From Newton 2nd law $F = ma$

$$W = mas$$

Using the equation of motion $v^2 = u^2 + 2as$. Making as the subject of the formula yield

$$as = \frac{1}{2}v^2 - \frac{1}{2}u^2$$

Substituting into the work equation above

$$W = m\left(\frac{1}{2}v^2 - \frac{1}{2}u^2\right) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

But $\frac{1}{2}mv^2 = KE$ i.e. the **kinetic energy**

$$W = KE_f - KE_i = \Delta KE$$

Work-energy theorem

Example

Example 4.2: The work done in accelerating a car

A 1000 kg car accelerates uniformly from rest to a speed of 30 m s^{-1} in a distance of 20 m. Determine

(a) the kinetic energy gained,

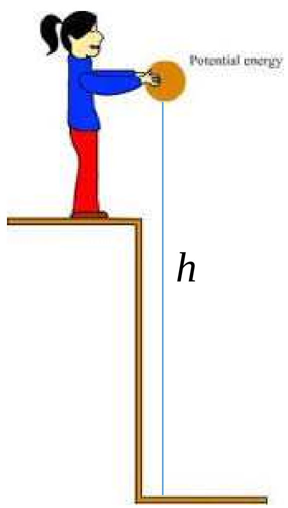
(b) the work done by the net force acting on the car, and

(c) the magnitude of the average net force.

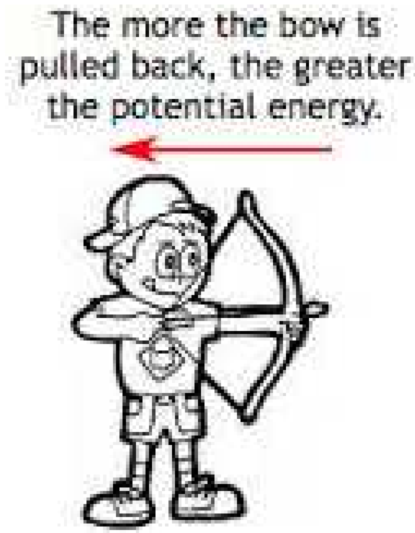
Potential energy

Potential energy is the energy possessed by an object due to its position. There are numerous types of potential energies like gravitational, elastic, electrical potential energy etc.

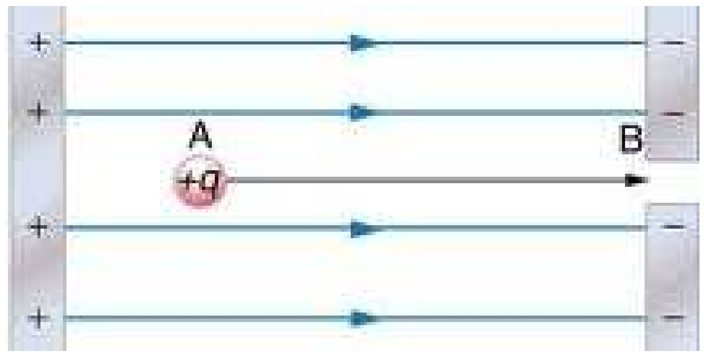
Gravitational Potential Energy



Elastic Potential Energy



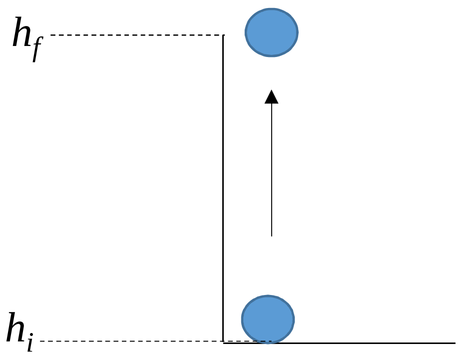
Electrical Potential Energy



We will concentrate on the gravitational potential energy. It is given by.

$$PE = mgh$$

Gravitational potential energy



If an object of mass *m* is raised by a vertical height $h = (h_f - h_i)$ in a gravitational field, then the work done against the gravitational force is equal to the change in gravitational potential energy of the object,

$$W = \Delta PE$$
$$W = PE_{Top} - PEb_{ottom}$$
$$W = mgh_f - mghi$$
$$W = mg(h_f - h_i)$$
$$W = mgh$$

Example 4.3: Work done in lifting an object

Find the work done in lifting a body whose mass is 5 kg through a vertical distance of 2 m.

Conservation of energy

The principle of conservation of energy
Energy may be transformed from one type to another without loss. In a closed or isolated system, the total amount of energy stays constant.

Mechanical energy

The mechanical energy of an object is defined as the sum of its potential and kinetic energies:

$$E = PE + KE.$$

Conservation mechanical energy

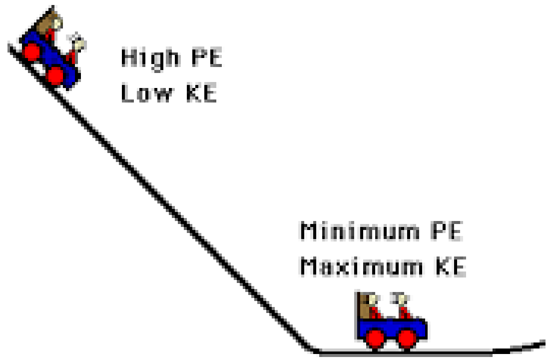
If there is no work done on an object by any **applied forces**, then the **mechanical energy** of the object is **conserved**. This means that the total mechanical energy of the object always remains the same. Hence ΔE = 0 and

$$\Delta PE + \Delta KE = 0$$

$$PE_f - PE_i + KE_f - KE_i = 0$$

$$PE_i + KE_i = PE_f + KE_f$$

$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$



Example 4.4: Conservation of mechanical energy

A mass of 80 kg slides down a smooth inclined plane 16 m high and 80 m long. Neglecting friction,

(a) calculate the potential energy of the mass at the top of the slope.

(b) How much kinetic energy does it have at the bottom of the slope?

(c) Determine the speed of the mass at the bottom of the slope.