PHYSICS 212
WAVES AND VIBRATIONS

13 Lectures
1 Test
In Exam: 45 out of 180 marks (25%)

Dr. S. Mthembu
P7 MSB, 2nd floor
X 5351
mthembus3@ukzn.ac.za

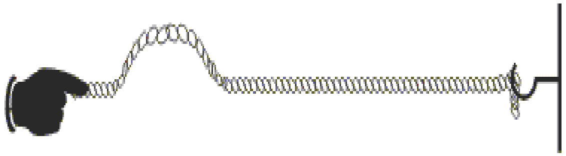
http://physicspmb.ukzn.ac.za/index.php/Courses

Mathematical description of wave motion

A wave can be described as a **propagation** of a **disturbance** through a **medium** from one location to another location. In a process, **energy** is transferred.

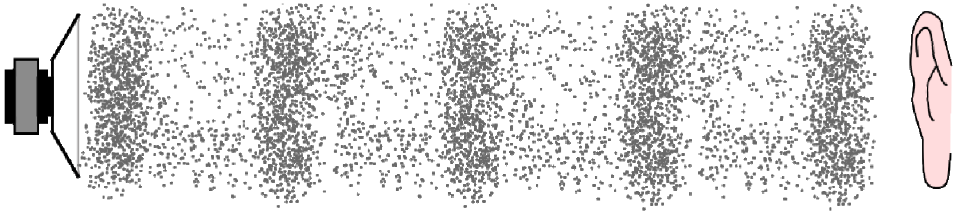
Examples of waves:

Wave on a string



Water wave

Sound wave



Common properties

- The disturbance travels through the medium
- Energy gets transferred to a distant point

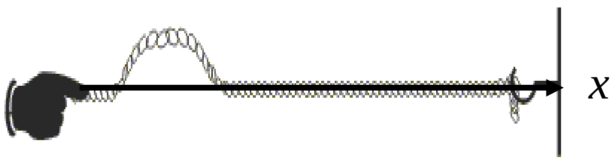
Mathematical description of wave motion

In this course, the disturbance in a wave will be described by **wave function $\boldsymbol{\phi}$**. The wave function could represents,

- An amplitude of the vibrating string,
- An elevation of water wave,
- An elevation of drum skin,
- Compression and rarefaction of air molecule,
- Magnitude of electric and magnetic field e.g. light, radio waves etc.

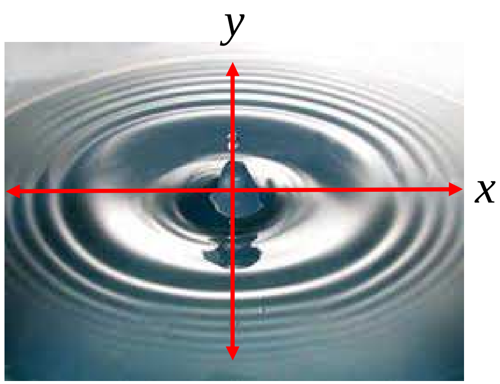
The position of the disturbance within the medium at any given **time** will be described by the **coordinates** in one-, two-, or three-dimensions, depending on the type of the medium. For example

String – one-dimension



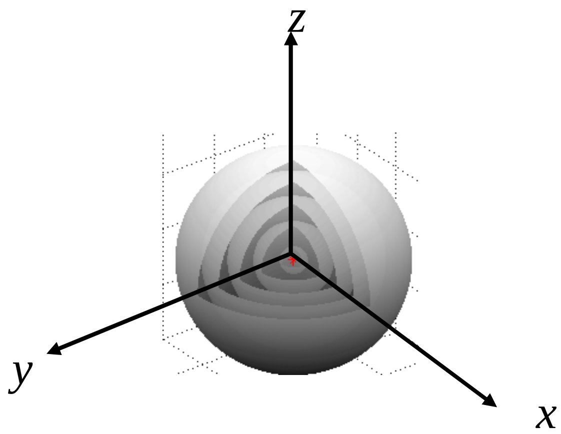
$\boldsymbol{\phi(x, t)}$

Water – two-dimensions



$\boldsymbol{\phi(x, y, t)}$

Air – three-dimensions



$\boldsymbol{\phi(x, y, z, t)}$

Mathematical description of wave motion

Waves that will be studied in this course are

- General one-dimensional wave
- Plane waves
- Spherical waves

All wave phenomena are governed by a **second order differential equation** called the **equation of Wave Motion**. A mathematical/theoretical treatment of waves involves solving these waves equation using the right boundary conditions and then interpret the solutions appropriately. The first theoretical treatment is to derive a one-dimensional wave equation.

Calculus technique required

Differentiation

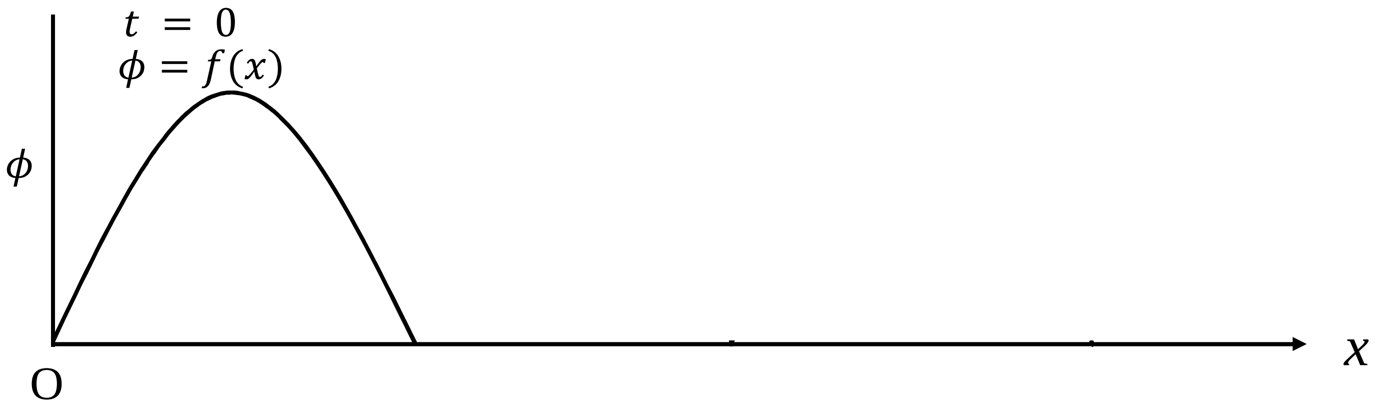
- By substitution
- Product rule
- Chain rule

Solving deferential equations

- Integration
- Separation of variables

Waves in one dimension: the non-dispersive wave equation

Consider a disturbance moving in a positive x-direction with a speed c. This disturbance is represented by a **wave function $\boldsymbol{\phi}$** and it could be anything like amplitude of the vibrating string, elevation of water wave, magnitude of electric field etc. The function ϕ is a function of position and time that is $\phi = \phi(x, t)$. If we consider a disturbance at $t = 0$, ϕ becomes a function of x alone, i.e. $\phi(x, 0) = f(x)$ which we call a wave profile. A wave profile moving at speed c without changing its shape i.e. no **dispersion** is shown in a diagram below,



After some time t, a wave profile has moved a distance $\mathrm{d} = ct$ from the origin O. Let's define a new origin O' be at $\mathrm{d} = ct$, and let the distance measured from O' be X. Thus a profile which describe a disturbance from O' is given by

$$\phi = f(X).$$

E.g. a green point has moved a distance X from O'.

With reference to O, the green point has moved a distance $x = ct + X$ which impels that $X = x - ct$. Thus

$$\boldsymbol{\phi = f(x - ct)}$$

is the wave profile describing a disturbance with reference to the original origin.

This is the most general expression of a **one-dimensional** wave moving at a **speed $\boldsymbol{c}$** to the **positive $\boldsymbol{x}$-direction without any change in shape (i.e. non-dispersive)**. The wave moving in negative direction is $\boldsymbol{\phi = f(x + ct)}$.

SM1

Waves in one dimension: the non-dispersive wave equation

f could be any function e.g. sin, cos, exp etc. For example

$$\phi(x,t) = sink(x - ct)$$

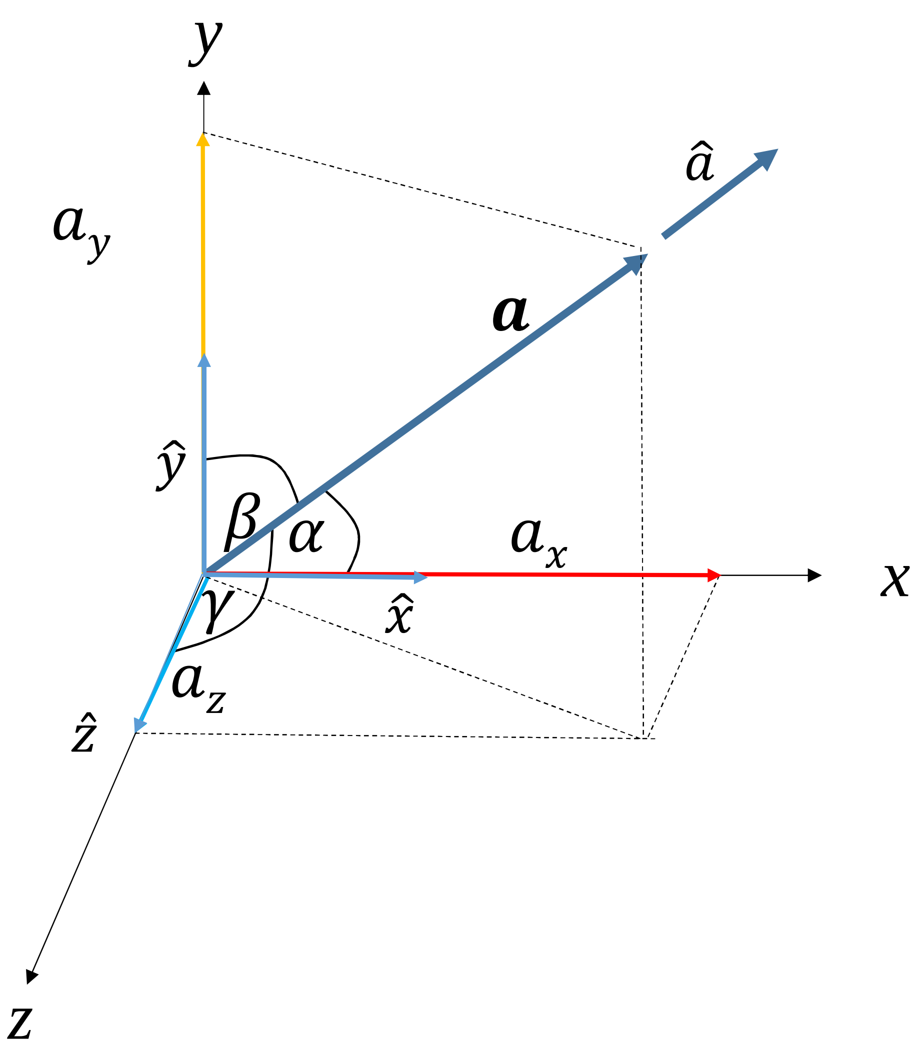
represents a sinusoidal disturbance.

Next we need to show that the expressions $\phi = f(x - ct)$ and $\phi = f(x + ct)$ satisfy a second order **differential equation**. To do this we will use a **substitution rule**.

Slide 6

SM1 Before proceding, do a quick and breif revision on calculus.
Sibusiso Mthembu, 2015/09/03

Direction cosine and a link to a unit vector



- $\hat{x}, \hat{y}$ and $\hat{z}$ are basis unit vectors of magnitude 1

- $\boldsymbol{a}$ is a vector such that $\boldsymbol{a} = |a|\hat{a}$
  $|a|$ is a magnitude of the vector and
  $\hat{a}$ is a unit vector indicating a direction

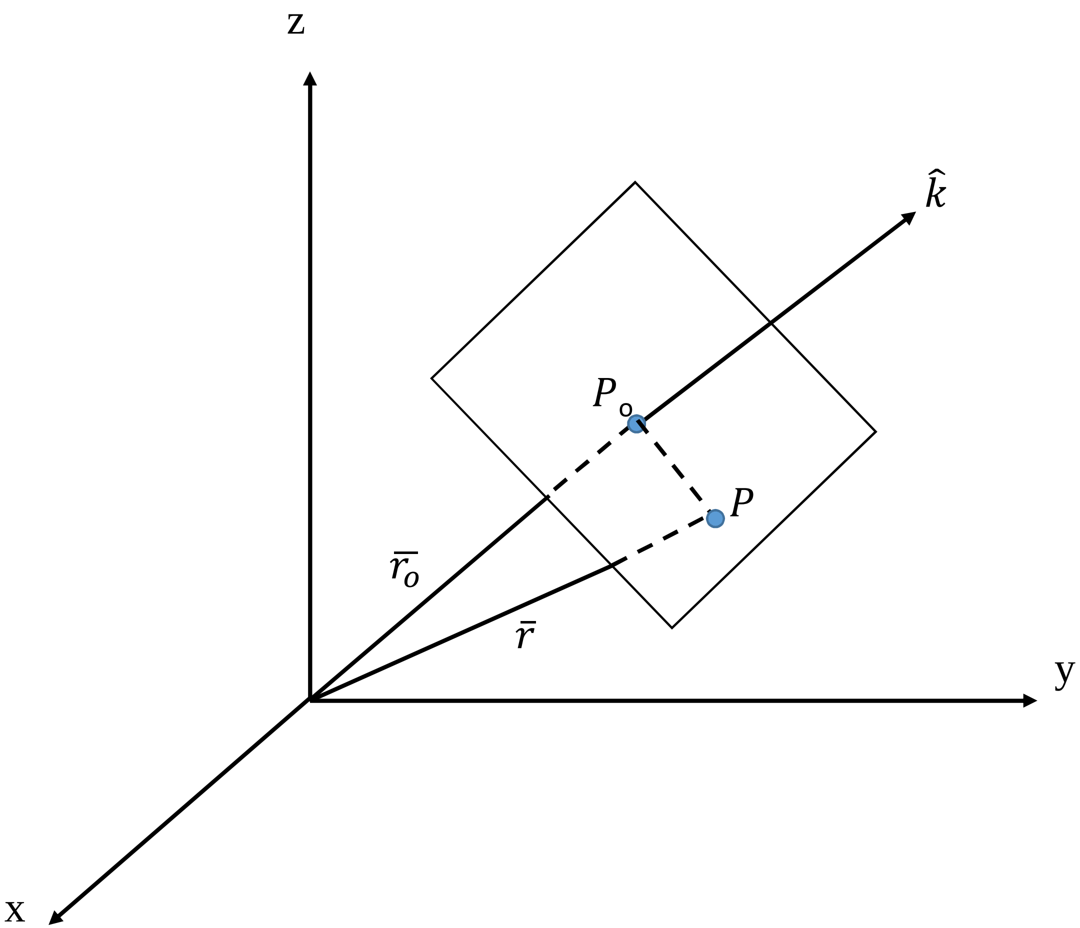
- a_x, a_y, a_z are x, y, z component of the vector

- α, β and γ are **direction angles**
- **Direction cosines** are cosine of direction angle i.e. $\cos\alpha, \cos\beta$ and $\cos\gamma$

What is the relationship between the **unit vector** and the **direction cosines.**

Equation of a plane

Consider a plane in a three-dimensional space. To determine an equation of a plane we need at least **a point on a plane** and **a normal vector to a plane**.



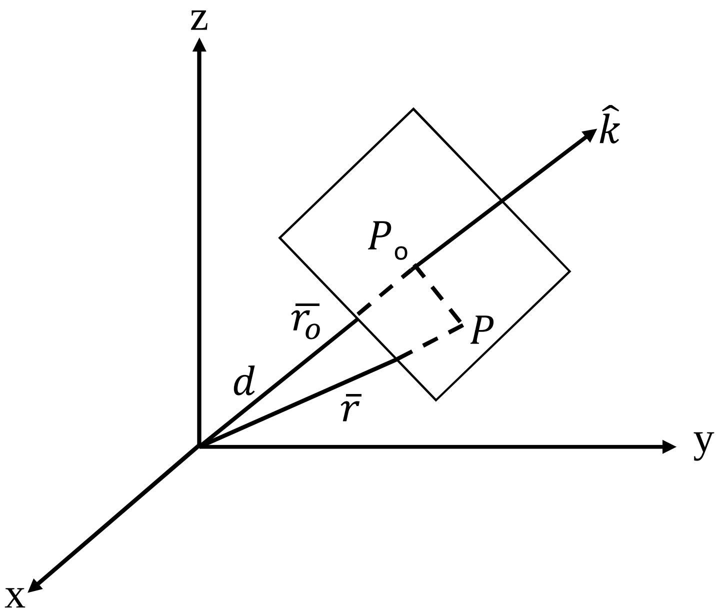
Assume that we are give a point $P_o = (x_o, yo, zo)$ and a normal vector $\hat{k} = (l, m, n)$, where l, m, n are direction cosines.

Assume we have another point $P = (x, y, z)$ on the plane.

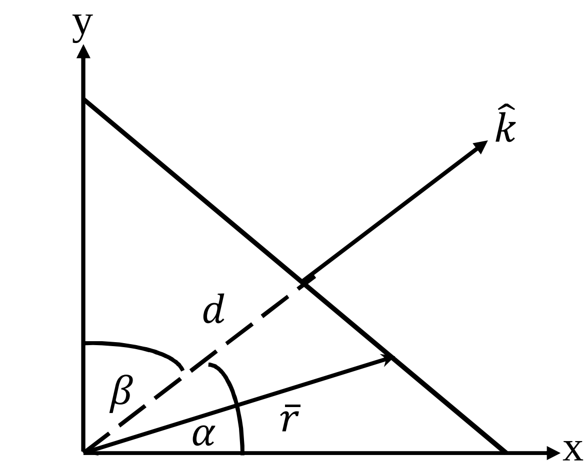
Define $\overline{r_o}$ and $\bar{r}$ to be position vectors of points P_o and P, and $|\overline{r_o}| = d$

Plane wave in three-dimensions

(a)

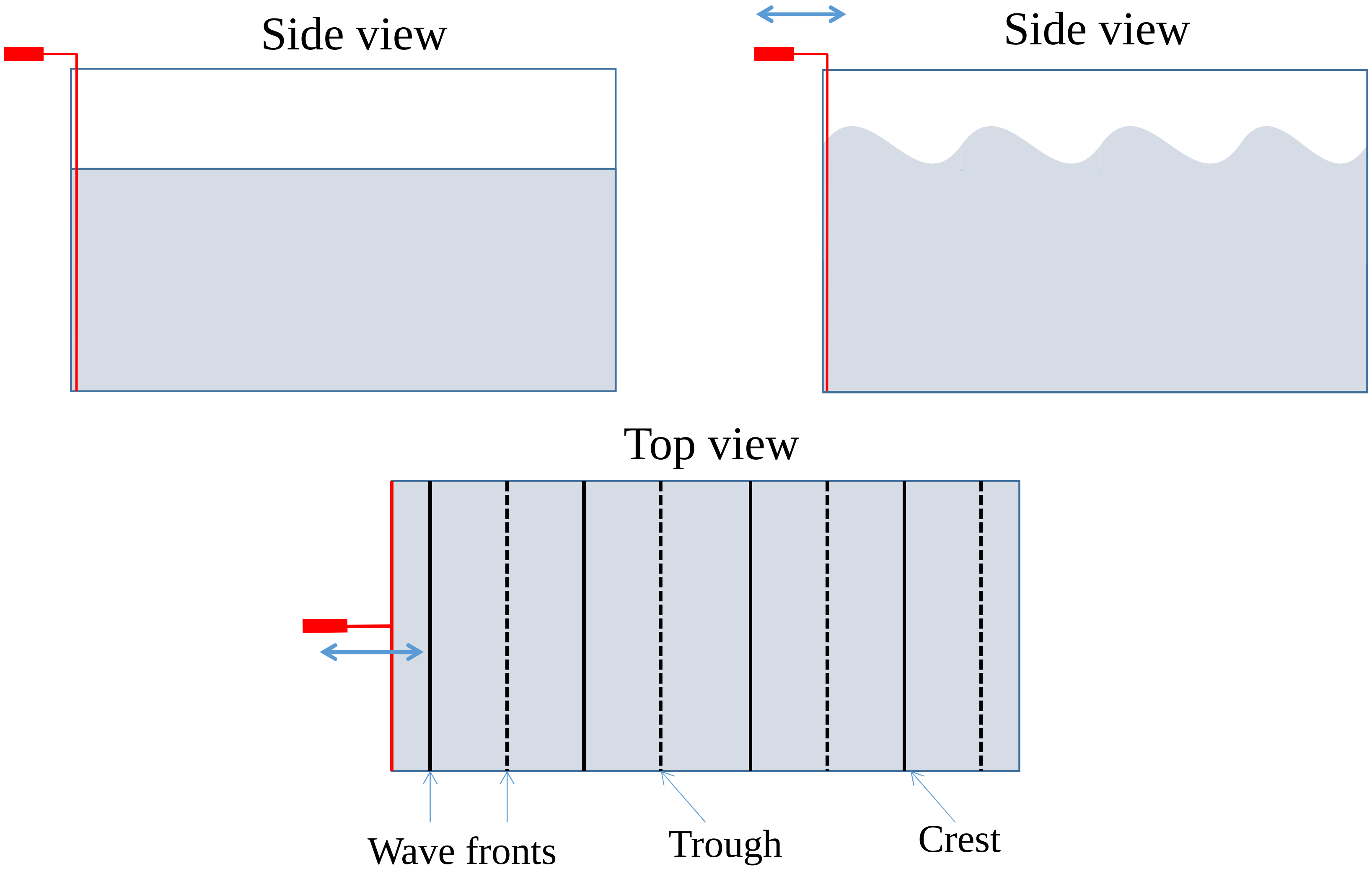


(b)

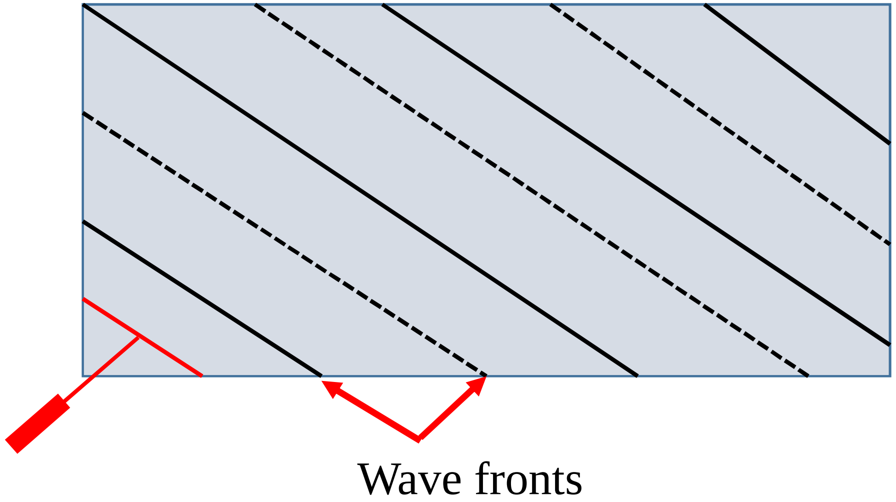


$\hat{k} = (l, m, 0), \qquad l = cos\alpha, m = cos\beta$

One-dimensional plane wave



Two-dimensional plane wave



Three-dimensional plane wave