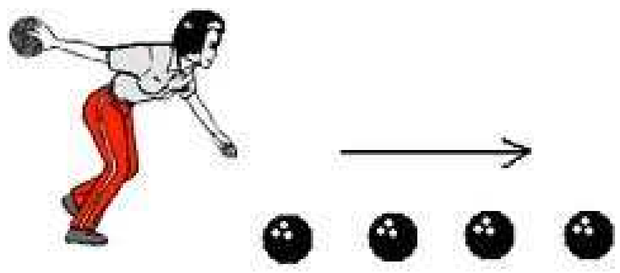
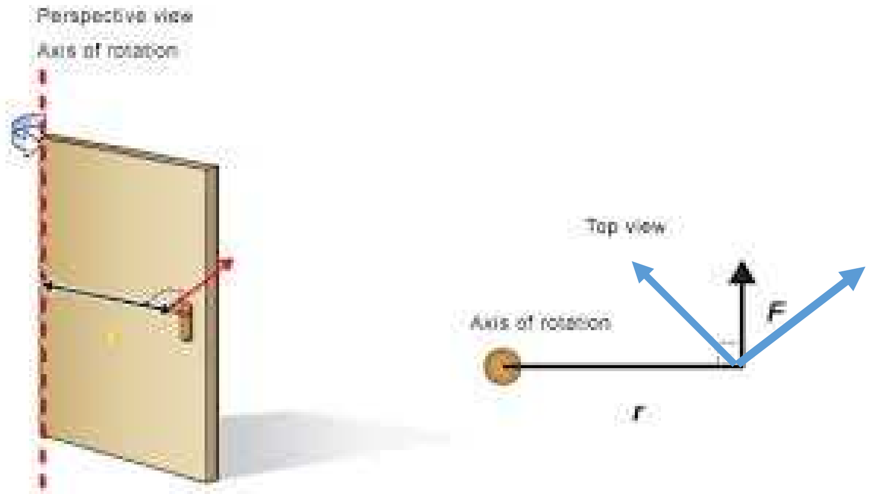
The moment of a force or the torque about an axis



The force is exerted on the ball, the ball moves from an **initial** position to **a final position**. The ball is said to have undergone a **translational** motion.



When the force is exerted on the door, it will rotate or turn. In this case the force produces a turning effect which is known as a **torque or moment of a force**. Thus, the torque τ of the force F about a give axis of rotation is given by a product of the force and a **perpendicular** distance from the axis of rotation. Mathematically,

$$\tau = F \times r$$

Units are Nm

The distance r is called a **lever arm** of the turning effect.

$$\tau \propto F$$
$$\tau \propto r$$

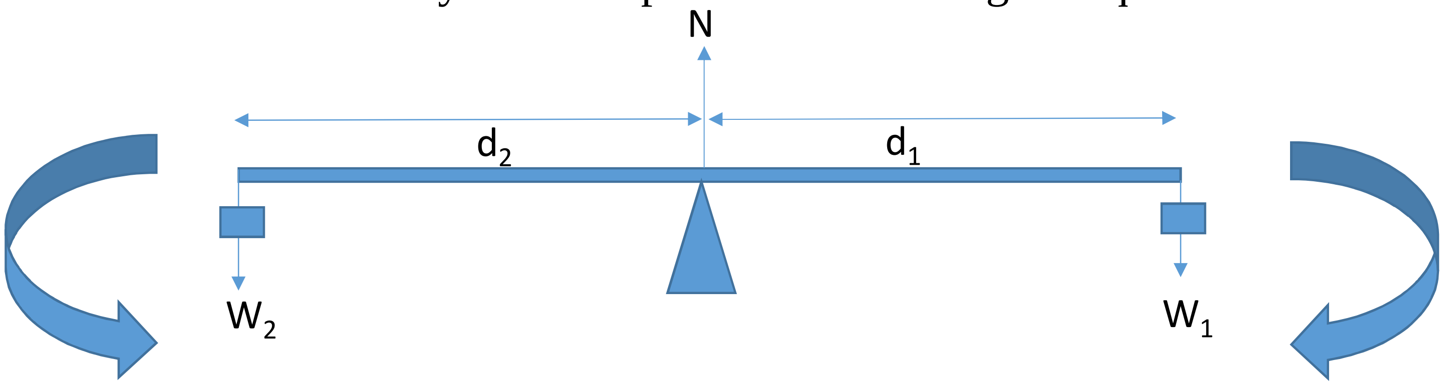
Application of torque

A mechanic is loosening a tight nut. He could ask a colleague to assist him exerting more force. That will increase a turning effect or a torque on a nut. Instead, he decided to use an extension pipe to increase the lever arm (r) and thereby increasing a torque (τ).

$$\tau = F \times r$$

Principle of moment

Consider a uniform bar resting on a triangular block, as shown in the figure below. Let the rotation axis or the pivot be at the point of contact between the bar and the triangular block, which is at the centre of the bar. Two objects are suspended at the ends of the bar and they have the potential of causing a torque.



$$\tau_2 = F_2 \times d_2$$

$$\tau_1 = F_1 \times d_1$$

The contention is that **anticlockwise** moments are **positive** and **clockwise** moments are **negative**

Principle of moments

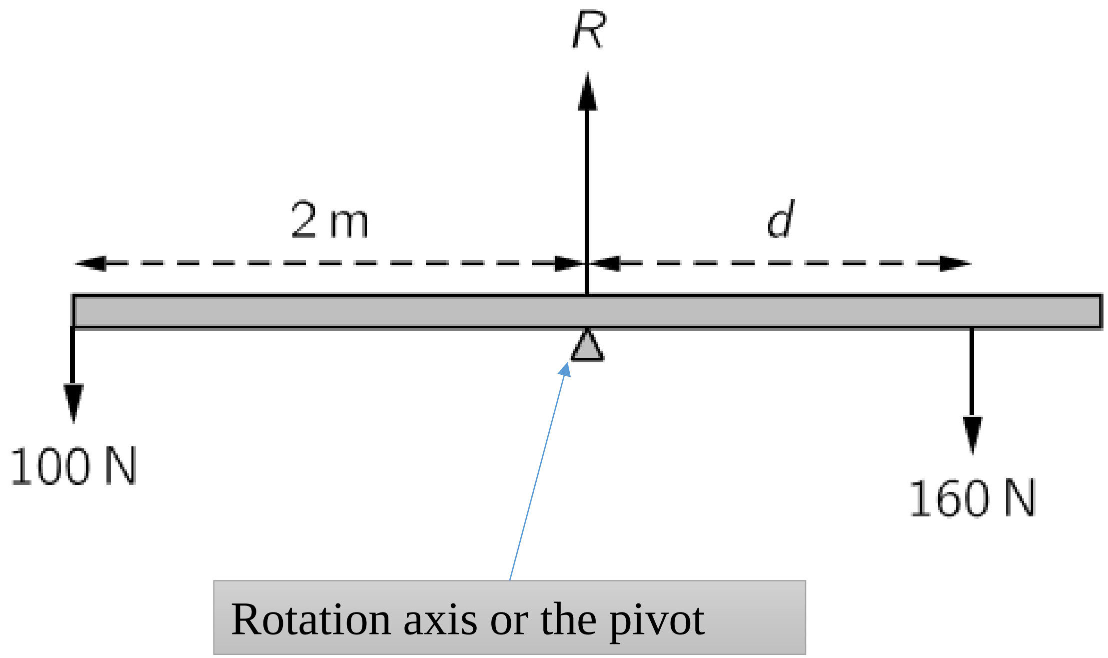
When a body is in equilibrium under the action of any number of coplanar forces, the algebraic sum of the moments of the forces about any point in the plane is zero. Mathematically,

$$\sum \tau = -\tau_1 + \tau_2 = -F_1 \times d_1 + F_2 \times d_2 = 0$$

Principle of moment

Example 3.11: Moment about an axis

Two children are playing on a see-saw. The total length of the see-saw is 4 m and it is pivoted exactly in the middle. One child weighs 100 N and sits at the end of one side of the see-saw. If the other child weighs 160 N, how far from the other end must she sit so that the see-saw is balanced.



The equilibrium of a rigid body under the action of a system of coplanar forces

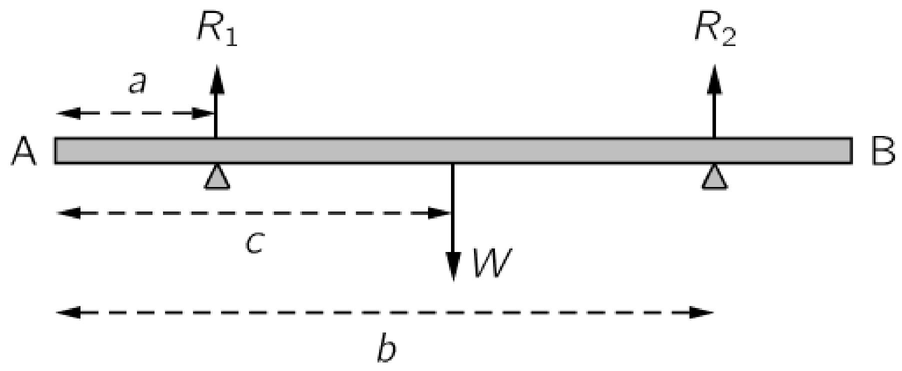
The figure shows a rigid bar AB resting on two supports. R_1 and R_2 are the reaction (normal) forces at the supports, and W is the weight of the bar. If the bar is uniform the weight acts at the centre of the bar. **Let the axis of rotation (pivot) be at point A**. For a rigid body at **equilibrium** (i.e. no acceleration),

1. The vector sum of all the external forces acting on the body must be zero:

$$\sum \boldsymbol{F} = \boldsymbol{0}, \text{ the considering } x\text{- and } y\text{-components, } \sum \boldsymbol{F_x} = \boldsymbol{0} \text{ and } \sum \boldsymbol{F_y} = \boldsymbol{0}$$

2. The algebraic sum of the moments of all the forces about any axis perpendicular to the plane of forces must be zero (Principle of moments):

$$\sum \boldsymbol{\tau} = \boldsymbol{0}$$



The net force is:

$$\sum F_r = R_1 + R_2 - W = 0$$

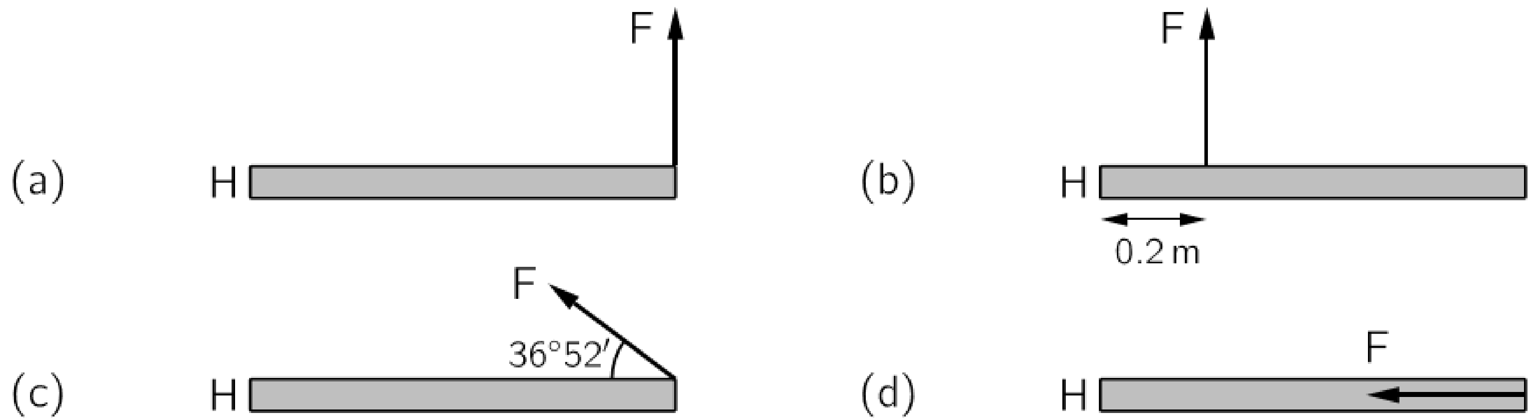
The moment or the torque about A:

$$\sum \tau = \tau_{Ant-clockwise} - \tau_{clockwise} = 0$$

$$\sum \tau = (R_1 \times a + R_2 \times b) - W \times c = 0$$

Example

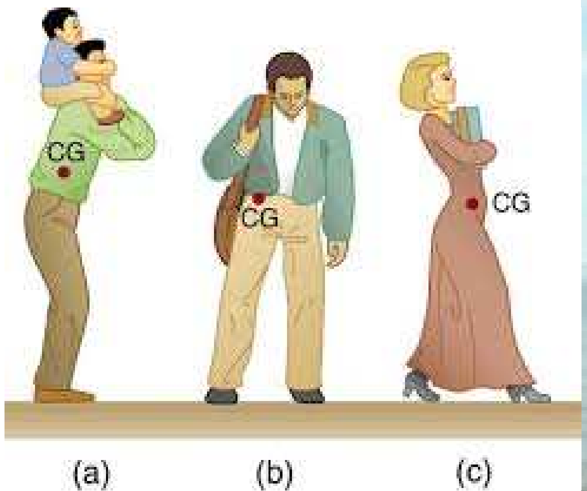
D8 A uniform rod 1m long and weighing 30 N is supported in a horizontal position on a pivot with weights of 40 N and 50 N suspended from its ends. Calculate the position of the pivot.



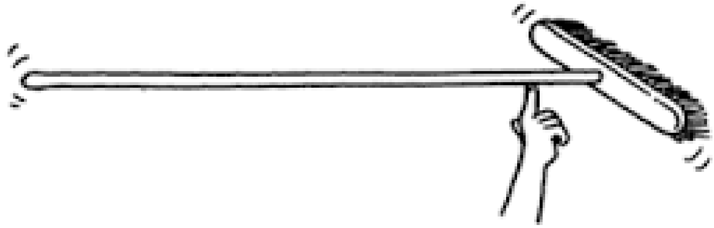
Centre of gravity

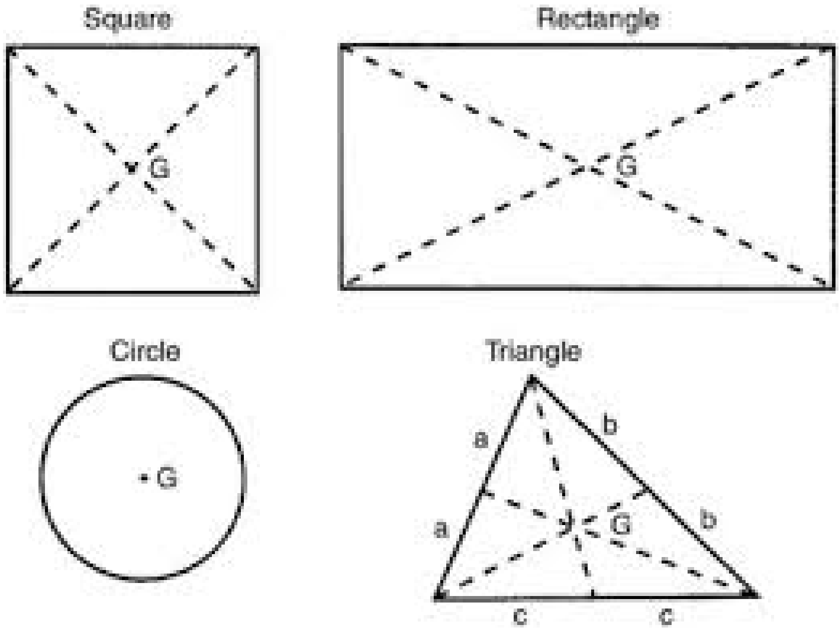
For any rigid body, a centre of gravity is defined as a point from which weight of the body may be considered to act. The weight around the centre of gravity is balanced.

Irregular objects

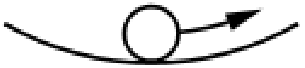


Regular objects

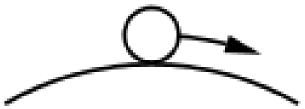




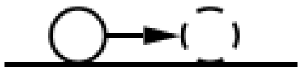
Stable, unstable and neutral equilibrium



Stable equilibrium: the ball returns to its original position.



Unstable equilibrium: the ball takes up a new position beyond an original small displacement.



Neutral equilibrium: The ball takes up a new position at the end of the displacement.