UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

SCHOOL OF CHEMISTRY & PHYSICS
JUNE 2013 MAIN EXAMINATION

PHYS361: QUANTUM MECHANICS AND CLASSICAL MECHANICS

TIME: 3 hours	**MAXIMUM MARKS:** 180

Internal Examiner(s):	**External Examiner:**
Dr Sergi	Dr D Naidoo (UNIVERSITY OF THE WITWATERSRAND)

General instructions:

- Answer **all** questions.
- As it is understanding that is being tested, explanation of the steps and physics involved must accompany any mathematics.
- Candidates are reminded to be as thorough as possible and to write legibly.

Section A: QUANTUM MECHANICS

1. What are the mathematical and physical characteristics of a wave function (specify the interpretation of the wave function)? (5)
2. Write the mathematical expression of a plane wave and give its physical interpretation. (4)
3. Show in detail an intuitive derivation of the time dependent Schrödinger equation in the case of a free particle. (10)
4. Consider a system of 2 electrons with charge $-e$ interacting with a nucleus with 2 protons of charge $+e$ and show, step by step, how to write explicitly the time independent Schrödinger equation of the system. (15)
5. Consider the Schrödinger equation for a particle of mass m in the case in which the potential does not depend on time: $V = V(\mathbf{r})$. Show in detail, explaining in words the relevant steps, how the time dependent Schrödinger equation can undergo a separation of variables. (20)
6. Consider a particle moving on a line, from $-\infty$ to $+\infty$, under the action of a potential which is non-zero and constant ($V = V_0$) only for $0 < x < a$. Find the form of the stationary wave function in the case in which the energy of the particle is less than the potential ($E < V_0$) and discuss the physical meaning of the solution. (20)
7. Solve the one-dimensional problem of the stationary motion of a quantum particle in an infinite potential well. (23)
8. Find the energy eigenvalues of the 1-dimensional harmonic oscillator using the algebraic method. (23)

(Total Marks [120])

Section B: Classical mechanics

1. Consider a system described by generalized coordinates q_i, $i = 1, \ldots, N$. Show how to derive the Euler-Lagrange equation from the Principle of Least Action. (20)

2. Consider the following Lagrangian density

$$\mathcal{L} = \frac{1}{2}\rho \left[\frac{\partial \phi(x,t)}{\partial t}\right]^2 - \frac{1}{2}T \left[\frac{\partial \phi(x,t)}{\partial x}\right]^2 ,$$

where ρ and T are constants.

(i) Derive the Euler-Lagrange equation from the Principle of Least Action. (20)

(ii) Consider the symmetry transformation $t \to t + \epsilon$ (where ϵ is constant) of the Lagrangian density $\mathcal{L}$ given above and derive the associated conservation law. (20)

(Total Marks: [60])