SCHOOL OF CHEMISTRY & PHYSICS
PIETERMARITZBURG CAMPUS
JUNE 2014 MAIN EXAMINATION
PHYS211: MECHANICS AND MODERN PHYSICS

TIME: 3 hours	**MAXIMUM MARKS:** 180

Internal Examiner(s):	**External Examiner or Moderating Board:**
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General instructions:

- ☞ Answer **any five (5)** questions.
- ☞ As it is understanding that is being tested, explanation of the steps and physics involved must accompany any mathematics.
- ☞ Candidates are reminded to be as thorough as possible and to write legibly.

Section A: Classical mechanics

Question 1

(a) Briefly explain what the following terms mean in the context of classical mechanics:

(i) inertial reference frame, (2)

(ii) initial conditions. (4)

(b) The acceleration a of a particle moving along the x-axis varies as

$$a = a_0 e^{-x/\lambda},$$

where a_0 and λ are positive constants. Suppose that at time $t = 0$ the particle is at the origin and moving with velocity $v_0 > 0$.

(i) Show that the velocity v is given by

$$v = v_0\sqrt{1 + \alpha(1 - e^{-x/\lambda})},$$

where α is a constant which must be clearly defined in your answer. (10)

(ii) Hence show that the position $x(t)$ of the particle is given in the inverse form by

$$t = \frac{1}{v_0}\int_0^{x(t)} \frac{du}{\sqrt{1 + \alpha(1 - e^{-u/\lambda})}}.$$

(2)

(iii) Neatly sketch $x(t)$ vs t for: ☞ infinite λ, ☞ finite λ. Clearly label each curve. (4)

(c) A particle of mass m, initially at the origin, is released from rest in a uniform gravitational field. The frictional force acting on the particle is $F_d = -mv/\tau$ where v is the velocity and τ is a characteristic time. Write down the equation of motion for the particle and then derive expressions for the velocity $v(t)$ and position $x(t)$ of the particle at time t in terms of τ and the acceleration due to gravity g. (14)

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Question 2

(a) A particle of mass m moving along the x axis is subject to a force $F = -kx$ where k is a positive constant.

(i) Write down the equation of motion of the particle. Show that the general solution to this equation, in terms of complex exponential functions, is

$$x(t) = a_1 e^{i\omega t} + a_2 e^{-i\omega t} \qquad \cdots \text{(I)}$$

where a_1 and a_2 are arbitrary constants. In your answer, define ω. (9)

(ii) Express (I) in the form $x(t) = A\cos(\omega t + \phi)$ where A and ϕ are constants. (4)

(iii) Show how to express A and ϕ in terms of the initial conditions x_0 and v_0. (5)

(b) A particle of mass m moves along the x-axis in a potential

$$V(x) = \tfrac{1}{2}kx^2 - \tfrac{1}{4}bx^4,$$

where k and b are positive constants.

(i) Sketch the graph of $V(x)$ versus x. Calculate the coordinates of the points of stable and unstable equilibrium, and identify these on your graph. (12)

(ii) Discuss the motion of a particle of mass m with energy $E \to 0$ oscillating about the point of stable equilibrium. (4)

(iii) What is the critical value of the energy E of the particle below which the potential can bind the particle? (2)

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Question 3

(a) A force $\mathbf{F}(\mathbf{r})$ acts on a particle of mass m.

(i) Prove the work-energy theorem

$$dK = \mathbf{F} \cdot d\mathbf{r},$$

where K is the kinetic energy of the particle. (4)

(ii) If $\mathbf{F}(\mathbf{r})$ is conservative, there exists a function $V(\mathbf{r})$ such that $\mathbf{F}(\mathbf{r}) = -\boldsymbol{\nabla} V$. Define the mechanical energy E of the particle and prove that E is conserved. (6)

(iii) Prove that the force $\mathbf{F} = -k(x, y, z)$, where k is a constant, is conservative and calculate the potential energy. (8)

(b) Consider a system of N particles subject to interparticle interactions and external forces.

(i) Write down the equations of motion. (Define all symbols used.) (6)

(ii) Use the equations of motions in (i) to show that

$$M\ddot{\mathbf{R}} = \mathbf{F}^{(e)}.$$

The meanings of the symbols M, $\mathbf{R}$ and $\mathbf{F}^{(e)}$ should be given in your answer. (12)

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Section B: Modern physics

(Where required, take c $= 2.998 \times 10^8\,\mathrm{m\,s^{-1}}$, $e = 1.602 \times 10^{-19}\,\mathrm{C}$, $\mathrm{m}_e = 9.109 \times 10^{-31}\,\mathrm{kg}$ and $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$.)

Question 4

(a) State Einstein's two postulates of the theory of special relativity. (4)

(b) (i) Consider two inertial frames S and S', whose axes conincide at time $t = 0$. Frame S' then moves away from frame S at constant velocity v, along the x,x' axes. Draw a labelled diagram of the frames S and S' showing the coordinates of a point P in the two systems. (4)

(ii) Write down the standard Lorentz transformation equations. (4)

(iii) Write down the expression for the factor γ of the Lorentz transformation. Using an appropriate sketch graph, discuss the limiting values of the factor γ. (4)

(c) (i) Show that the x component of the velocity of a particle in an inertial frame S' moving at speed v along the x direction with respect to frame S is given by (5)

$$u'_x = \frac{u_x - v}{1 - u_x \frac{v}{c^2}}.$$

(ii) An unstable particle at rest in the laboratory frame splits into two identical pieces, which fly apart in opposite directions at Lorentz factor $\gamma = \frac{13}{5}$ relative to the laboratory frame. What is the relative speed of one particle with respect to the other? (10)

(d) The speed and kinetic energy of a particle were measured to be $1.64 \times 10^8\,\mathrm{m\,s^{-1}}$ and 99.5 keV respectively. Determine the rest mass of the particle. (5)

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Question 5

(a) (i) Write down an expression for the relativistic kinetic energy of a particle and show that in the non-relativistic limit this expression reduces to $K = \frac{1}{2}m_0 v^2$. (4)

(ii) Derive the expression $E^2 = m_0^2 c^4 + p^2 c^2$ for a free particle. (6)

(b) Describe the photo-electric effect and summarize the observed features of this effect. Show how some of these features cannot be explained with the wave model of electromagnetic radiation, but that they are readily explained using the Einstein photon hypothesis. (12)

(c) (i) Draw a labelled diagram of a collision between a photon and a stationary electron to illustrate Compton scattering. Write down the equations that ensure conservation of energy and momentum. (You do not need to derive the equation for the Compton effect.)

(10)

(ii) Compton verified the shift of wavelength of scattered photons experimentally by scattering x-rays from a graphite target. Explain why x-rays were used instead of visible light. Also explain why there were two peaks in the wavelength of the scattered photons in the experimental results. (4)

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Question 6

(a) Describe the Davisson Germer experiment which confirmed the existence of de Broglie waves. Illustrate your arguments with the use of the following data: Electrons accelerated through an electrostatic potential of 54 V gave a first-order maximum at a glancing angle of 65° from a crystal with a plane spacing of 0.091 nm. (14)

(b) With the aid of the de Broglie hypothesis and the assumption that standing waves lead to stable states, obtain

(i) the energy level formula

$$E_n = \frac{n^2 h^2}{8mL^2}\,, \quad n = 1, 2, 3, \ldots$$

for a particle of mass m in a one-dimensional infinite square well of width L, and (6)

(ii) the quantization condition for angular momentum

$$mvr = \frac{nh}{2\pi}\,, \quad n = 1, 2, 3, \ldots$$

of an electron in a hydrogen atom. (4)

(c) (i) Draw up a table showing the allowed sets of quantum numbers (n, ℓ, m_ℓ, m_s) of a hydrogen atom for $n = 1, 2, 3$. (5)

(ii) State Pauli's exclusion principle and briefly discuss the relevance of these sets in the treatment of many-electron atoms. (4)

(iii) He, Ne and Ar have 2, 10 and 18 electrons respectively. Use your table to suggest why these atoms are chemically inert. (3)

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