Capacitance And Gauss' Law

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Storing Energy

Pulling back on an archer's bow or stretching a spring is storing mechanical energy as elastic potential energy.

A capacitor is a device that stores *electric* potential energy, as well as storing charge.

The energy stored a charged capacitor is related to the electric field inside of the device.

The electric potential energy can be regarded as stored in the electric field itself.

Capacitors

Capacitors are devices that store charge.
A capacitor consists of two conductors separated by some (electrically) insulating material.
(Consider the parallel plates arrangement discussed previously.)
Charge is transferred from one conductor (plate) to the other, so that one plate has a net negative charge while the other has an equal amount of net positive charge.
This results in a potential difference between the conductors. Work must be done to move charges through the potential difference, and this work is stored as electric potential energy.
Capacitors are used in the flash units of cameras, in air bags, in lasers and in radio/TV receivers.

Capcitance

The charge stored on each conductor is proportional to the potential difference between the conductors. The proportionality constant is called the capacitance.
The capacitance C of a capacitor is defined as

$$C = \frac{q}{\Delta V}$$

where q is the magnitude of the charge stored by the capacitor, and ΔV is the potential difference between the conductors.
The SI unit of capacitance is the farad, F (1 F = 1 C V^{-1}).
Note that 1 F is a large value of capacitance, so typical values are µF, nF (or even pF).

Components In An Electrical Circuit

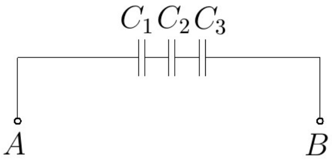
Capacitors are made in standard capacitance values for a range of voltages.

To obtain a specific capacitance for a given application, combinations of capacitors can be used. Two combinations of capacitors in a circuit are the *series* and *parallel* combinations.

An *equivalent* capacitance can be found for each combination.

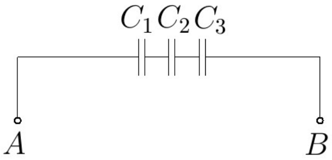
Capacitors In Series

A series combination of capacitors is one where the capacitors are connected one after the other by conducting wires.



A constant potential difference across the ends of the combination (across points A and B) results in the charging of each capacitor. The magnitude of the charge on *each* plate (conductor) on *every* capacitor must be the same.

Capacitors In Series



Positive charge accumulates at the left-hand side plate of C_1.
An equivalent amount of negative charge accumulates on the right-hand side plate of C_3.
Negative and positive charges arrange themselves so that there is the same amount of charge on each plate.

Capacitors In Series

The potential difference ΔV across the combination is equal to the sum of the potential differences across each capacitor.

$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$$

Since the magnitude of the charge on each capacitor in series is the same, from the definition of capacitance

$$\begin{aligned} \frac{q}{C_{\text{eq}}} &= \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} \\ \therefore \frac{1}{C_{\text{eq}}} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \end{aligned}$$

where C_{eq} is the equivalent capacitance for the combination.

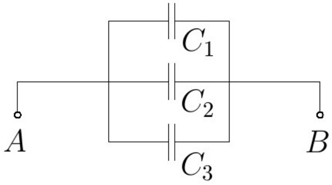
Two Capacitors In Series

For the special case of two capacitors in series, the product-sum rule can be used.

$$C_{\text{eq}} = \frac{C_1 \times C_2}{C_1 + C_2}$$

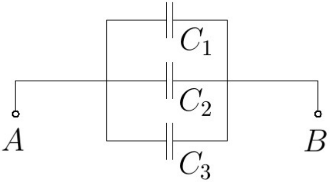
Capacitors In Parallel

A parallel combination of capacitors is



The potential difference ΔV across points A and B must be the same potential difference across each of the three branches (and hence each of the three capacitors).
The total charge Q is equal to the sum of the charges on each capacitor. (Note that the charges on each of the capacitor in this case need not be the same.)

Capacitors In Parallel



The total charge is

$$Q = q_1 + q_2 + q_3 = C_1\Delta V_1 + C_2\Delta V_2 + C_3\Delta V_3$$

The total charge Q is given by the equivalent capacitance C_{eq}, and since the potential difference for each capacitor is the same

$$\begin{aligned} C_{\text{eq}}\Delta V &= C_1\Delta V + C_2\Delta V + C_3\Delta V \\ C_{\text{eq}} &= C_1 + C_2 + C_3 \end{aligned}$$

Energy Stored In A Capacitor

The electric potential energy stored in a charged capacitor is equal to the work done in order to charge the capacitor.

The work done in charging a capacitor is given by $W = q\overline{\Delta V}$, where q is the charge on the capacitor and $\overline{\Delta V}$ is the *average* potential difference across the capacitor (during charging).

For a final potential difference ΔV, the work done to store the capacitor is

$$W = q\overline{\Delta V} = \frac{1}{2}q\Delta V$$

The work done is equal to the energy stored in the capacitor, if the potential energy of the uncharged capacitor is taken as zero.

$$W = U = \frac{1}{2}q\Delta V = \frac{1}{2}C(\Delta V)^2 = \frac{q^2}{2C}$$

Electric Flux

Given a point charge in at a particular point in space, what is the electric field due to that charge in the region of space around it?
If the electric field pattern in a region of space is known, what is the charge (charge distribution) that is generating the electric field?
The electric flux ψ of an electric field **E** through a small closed surface area A is defined as

$$\psi = (E \cos \theta) A$$

$(E \cos \theta)$ is the component of the flux perpendicular to the area A.

Gauss' Law

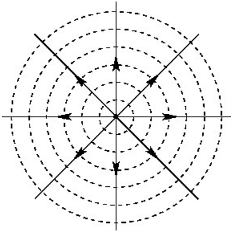
Gauss' Law states that the electric flux through any closed surface is proportional to the algebraic sum of all of the charges enclosed in that surface.

$$\psi \propto Q_{\text{enc}} = \frac{1}{\varepsilon_0} Q_{\text{enc}}$$

where Q_{enc} is the charg enclosed by the surface.
Gauss' Law is one of the fundamental laws of electromagnetism.

Gauss' Law

Gauss' Law can be used to find the electric field around a charge distribution, for example a point charge.



The closed surface is taken to be the surface area of a sphere. By Gauss' Law

$$\frac{1}{\varepsilon_0} Q_{\text{enc}} = E \times (4\pi r^2) \;\Rightarrow\; E = \frac{1}{4\pi\varepsilon_0} \frac{Q_{\text{enc}}}{r^2}$$