UNIVERSITY OF KWAZULU-NATAL

SECOND SEMESTER EXAMINATIONS: NOVEMBER 2007

SUBJECT AND COURSE: PHYSICS HONOURS 742

TIME: 2 hours **TOTAL MARKS:** 120

Internal Examiners: Professor O L de Lange
External Examiner: Dr R de Mello Koch (WITS UNIVERSITY)

GENERAL INSTRUCTIONS

- ☞ Answer **TWO QUESTIONS** from Section A and **ONE QUESTION** from Section B.
- ☞ This paper consists of **3** pages, please ensure that you have them all.
- ☞ Explanation of the steps and physics involved must accompany any mathematics.
- ☞ Candidates are reminded to be as thorough as possible and to write legibly.

SECTION A: STATISTICAL PHYSICS

Question 1

(a) State the physical conditions which specify (i) the microcanonical ensemble, (ii) the canonical ensemble, (iii) the grand canonical ensemble. Which physical quantities are fixed in each case? (12)

(b) Starting with the Gibbs distribution of the grand canonical ensemble, and using (without proof) appropriate results from the theory of fluctuations, show how, in the thermodynamic limit,

(i) the grand canonical ensemble reduces to the canonical ensemble,

(ii) the canonical ensemble reduces to the microcanonical ensemble. (28)

[40]

Question 2

(a) Consider a perfect monatomic gas consisting of N atoms, each of mass M, in a volume V.

(i) Show that the partition function for a single atom is

$$\xi = V(2\pi MkT/h^2)^{\frac{3}{2}}\,.$$

(The following results should be assumed: The energy levels:

$$\varepsilon = \frac{h^2}{8\,MV^{\frac{2}{3}}}\;(\ell^2 + m^2 + n^2)$$

(Question 2(a)(i)... continued over the page)

where ℓ, m and n = 1, 2, ...

$$\int_0^\infty e^{-\alpha x^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}\ .)$$

(10)

(ii) Given the expression $Z = \xi^N/N!$ for the partition function of the gas in the classical limit, show that the entropy and chemical potential of the gas in this limit are

$$S = \frac{5}{2}Nk + Nk\ell n\frac{V}{N}\left(\frac{2\pi MkT}{h^2}\right)^{\frac{3}{2}} \qquad \text{(I)}$$

and

$$\mu = kT\ell n\frac{N}{V}\left(\frac{h^2}{2\pi MkT}\right)^{\frac{3}{2}} \qquad \text{(II)}$$

(Ignore any contribution from the electronic states.) (12)

(iii) Explain what is meant by intensive and extensive variables. Deduce from (I) and (II) that S is extensive and μ is intensive. (10)

(b) (i) Explain what is meant by the classical limit of the Bose–Einstein and Fermi–Dirac distributions. (3)

(ii) Define the quantum volume V_Q and explain briefly its significance. Use the result (II) above to show that the condition for the classical limit is

$$\frac{V}{N} >> V_Q\ .$$

(5)

[40]

Question 3

(a) Consider the wave equation

$$(\nabla^2 + \mathbf{k}^2)\Psi = 0$$

where $\mathbf{k} = (k_x, k_y, k_z)$ is a constant. Show that the number of standing wave solutions having a value of k between k and $k + dk$ in a cube of volume V is

$$D(k)dk = \frac{Vk^2}{2\pi^3}dk\ .$$

(8)

(b) Explain briefly what is meant by black–body radiation. (4)

(c) Starting with the Bose–Einstein distribution

$$\langle n_j \rangle = \frac{1}{e^{\frac{\varepsilon_j - \mu}{kT}} - 1}\ ,$$

and explaining your reasoning, deduce Planck's formula for black–body radiation

$$dU = \frac{V\hbar}{\pi^2 c^3}\,\frac{\omega^3 d\omega}{e^{\frac{\hbar\omega}{kT}} - 1}$$

where the symbols have their usual meaning. (20)

(Question 3(d)... continues over the page)

(d) Deduce

- (i) The Rayleigh–Jeans formula. (2)
- (ii) Wien's formula. (2)
- (iii) The Stefan–Boltzmann law. (4)

(Use the result

$$\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15} .)$$

[40]

SECTION B: SUPERCONDUCTIVITY

Question 4

(a) Write notes on the following topics:

- (i) The critical magnetic field and the critical current of type I superconductors. (6)
- (ii) The Meissner effect and screening currents. (5)
- (ii) Trapped flux and persistent currents. (5)

(b) Write a short essay (approximately two pages) on type II superconductivity. (24)

(Where appropriate, illustrate your answer with suitable diagrams.) [40]

Question 5

(a) Starting with the expression for probability current density

$$\mathbf{j} = \frac{\hbar}{2im}(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) - \frac{e}{m}\mathbf{A}|\Psi|^2 \ ,$$

and stating any assumptions you make, give a justification for the two London equations

$$\Lambda \text{ curl } \mathbf{j}_s = -\mathbf{B}$$

and

$$\Lambda \partial \mathbf{j}_s / \partial t = \mathbf{E}$$

(Your answer should include a definition of all symbols used.) (12)

(b) Deduce the following consequences of the London equations:

- (i) The Meissner effect. (You may consider a convenient geometry, such as a superconducting half–space.) (14)
- (ii) Fluxoid conservation. (8)
- (iii) Fluxoid quantization. (6)

[40]