UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

School of Chemistry & Physics
Pietermaritzburg Campus
June 2015: Main Examination
PHYS211: Mechanics & Modern Physics

TIME: 3 hours **MAXIMUM MARKS:** 180

Internal Examiner:	**External Examiner or Moderating Board:**
Dr. A. Welter	Prof. G. T. Mola (Pietermaritzburg Campus)

General instructions

1. Answer **any five (5)** questions.
2. As it is understanding that is being tested, explanation of the steps and physics involved must accompany any mathematics.
3. Candidates are reminded to be as thorough as possible and to write legibly.
4. It is the candidate's responsibility to ensure that this paper has **5 numbered pages**.
5. Marks have been allocated in such a way that 1 mark corresponds approximately to one minute. Candidates are advised not to spend a disproportionate amount of time on any question.

SECTION A: Classical mechanics

Question 1

(a) A force $F(t) = \alpha - \beta t$, where α and β are positive constants, acts on a particle of mass m which moves along the x axis. At time $t = 0$ the particle is at rest at the origin.

(i) Write down the equation of motion of the particle. Integrate this equation to determine the velocity $v(t)$ and poistion $x(t)$ in terms of m, α and β. (9)

(ii) Sketch the graphs of $F(t)$ vs t and $v(t)$ vs t for $t \geq 0$ and label all significant points. (9)

(b) A particle of mass m, initially at the origin, is released from rest in a uniform gravitational field. The frictional force acting on the particle is $F_{\mathrm{d}} = -mv/\tau$ where v is the velocity and τ is a characteristic time.

(i) Write down the equation of motion for the particle. (4)

(ii) Derive an expression for the velocity $v(t)$ of the particle at time t in terms of τ and the acceleration due to gravity g. What is the terminal velocity of the particle? (9)

(iii) Sketch the graph of $v(t)$ vs t showing the asymptotic behaviour of $v(t)$. (5)

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Question 2

(a) A particle of mass m moving along the x axis is subject to a restoring force $F = -kx$ where k is a positive constant.

(i) Write down the equation of motion of the particle. Show that the general solution to this equation, in terms of complex exponential functions, is

$$x(t) = a_1 e^{i\omega t} + a_2 e^{-i\omega t},$$

where a_1 and a_2 are arbitrary constants. In your answer, define ω. (9)

(ii) Express the general solution obtained in (i) in the form $x(t) = A\cos(\omega t + \phi)$, where A and ϕ are constants. (8)

(iii) Show how to express A and ϕ in terms of the initial conditions ($t = 0$) x_0 and v_0. (5)

(b) A particle of mass m moves along the x axis in a potential

$$V(x) = \tfrac{1}{2}kx^2 - \tfrac{1}{4}bx^4,$$

where k and b are positive constants.

(i) Sketch the graph of $V(x)$ versus x. Calculate the coordinates of the points of stable and unstable equilibrium, and identify these on your graph. (12)

(iii) What is the critical value of the energy E of the particle below which the potential can bind the particle? (2)

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Question 3

(a) A force $\mathbf{F}(\mathbf{r})$ acts on a particle of mass m.

(i) Prove the work-energy theorem

$$dK = \mathbf{F} \cdot d\mathbf{r},$$

where K is the kinetic energy of the particle. (4)

(ii) If $\mathbf{F}(\mathbf{r})$ is conservative, there exists a function $V(\mathbf{r})$ such that $\mathbf{F}(\mathbf{r}) = -\boldsymbol{\nabla} V$. Define the mechanical energy E of the particle and prove that E is conserved. (6)

(iii) Prove that the force $\mathbf{F} = -k(x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}})$, where k is a constant, is conservative and calculate the potential energy. (8)

(b) Consider a system of N particles subject to inter-particle interactions and external forces.

(i) Write down the equations of motion. (Define all symbols used.) (6)

(ii) Use the equations of motions in (i) to show that

$$M\ddot{\mathbf{R}} = \mathbf{F}^{(e)}.$$

The meanings of the symbols M, $\mathbf{R}$ and $\mathbf{F}^{(e)}$ should be given in your answer. (12)

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SECTION B: Modern physics

Where required, take $c = 2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$, $e = 1.602 \times 10^{-19}\,\mathrm{C}$, $m_e = 9.109 \times 10^{-31}\,\mathrm{kg}$ and $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$.

Question 4

(a) State Einsteins two postulates of the theory of special relativity. (4)

(b) (i) Consider two inertial frames S and S', whose axes coincide at time $t = 0$. Frame S' then moves away from frame S at constant velocity v, along the x,x' axes. Draw a labelled diagram of the frames S and S' showing the coordinates of a point P in the two systems. (4)

(ii) Write down the standard Lorentz transformation equations. (3)

(iii) Write down the expression for the factor γ of the Lorentz transformation. Using an appropriate sketch graph, discuss the limiting values of the factor γ. (4)

(c) Show that the x component of the velocity of a particle in an inertial frame S' moving at speed v along the x direction with respect to frame S is given by (5)

$$u'_x = \frac{u_x - v}{1 - u_x \frac{v}{c^2}}.$$

(d) An unstable particle at rest in the laboratory frame splits into two identical pieces, which fly apart in opposite directions at Lorentz factor $\gamma = 25/7$ relative to the laboratory frame. What is the relative speed of one particle with respect to the other? Express your answer to 5 significant figures. (12)

(e) Determine the speed of a particle whose mass is five times its rest mass. (4)

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Question 5

(a) Derive the expression $E^2 = m_0^2 c^4 + p^2 c^2$ for a free particle. (6)

(b) (i) Describe the photoelectric effect and summarize the observed features of this effect. Show how some of these features cannot be explained with the wave model of electromagnetic radiation, but that they are readily explained using the Einstein photon hypothesis. (11)

(ii) Prove that the photoelectric effect cannot occur for free electrons. (4)

(c) A 5 MeV electron undergoes annihilation with a positron that is at rest, producing two photons. One of the photons travels in the direction of the incident electron. Calculate the energy of each photon. (Hint: Consider the emission of the second photon both in the same, as well as the opposite, direction of the first.) (15)

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Question 6

(a) (i) Draw a labelled sketch graph showing the spectral distribution of a typical black-body radiator as a function of frequency. On the same axes, draw the distribution predicted by the Rayleigh–Jeans theory. (4)

(ii) Explain what is meant by the 'ultraviolet catastrophe'. What assumptions did Planck make that allowed him to deduce his radiation formula. (6)

(b) State de Broglie's hypothesis. (2)

(c) With the aid of the de Broglie hypothesis and the assumption that standing waves lead to stable states, obtain

(i) the energy level formula

$$E_n = \frac{n^2 h^2}{8mL^2} \qquad n = 1, 2, 3, \ldots$$

for a particle of mass m in a one-dimensional infinite square well of width L, and (6)

(ii) the quantization condition for angular momentum

$$mvr = \frac{nh}{2\pi} \qquad n = 1, 2, 3, \ldots$$

of an electron in a hydrogen atom. (4)

(d) Determine the accelerating potential necessary to give an electron a de Broglie wavelength of 1 Å (which is of the order of the size of the inter-atomic spacing of atoms in a crystal). Explain why a non-relativistic approach to the calculation is justified. (6)

(e) Give the names, symbols and allowed values of the quantum numbers that specify the state of an electron in multi-electron atoms. (8)

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