Fluids at rest - properties

A fluid is any substance that can flow, including liquids, gases. In this course we will concentrate on the liquids. We will explore three properties of liquids i.e. **density**, **pressure** and **upthrust**, when they are at **rest (Fluid statics)**. While gasses can be compressed, liquids are **incompressible**.

Density

The density (ρ) of a substance is its mass per unit volume, mathematically it is given by

$$\rho = \frac{m}{V}$$

and unit is kg m^{-3}.

The density of a substance changes with temperature due to expansion or contraction. Therefore when the density of a substance is given, the temperature should also be given. The density of pure water at 4° C is 1000 kg per cubic metre, i.e. 1000 kg m^{-3}. Objects made up of the same material have the same density even though their sizes and shapes are different. Objects in the figure below are all made up of steel, thus their density is the same.

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Pressure

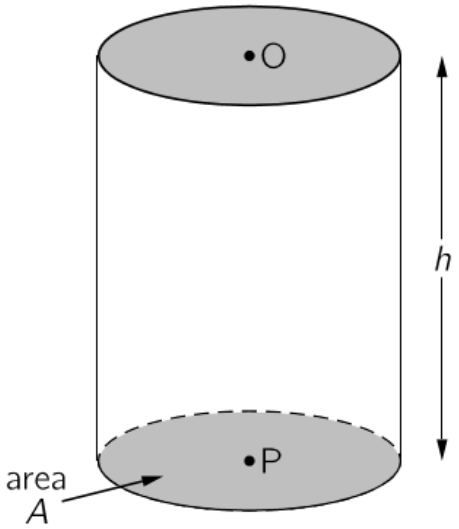
Is defined as a force F exerted acts over an area A perpendicular to the force, mathematically it is given as

$$P = \frac{F}{A}$$

and the units of pressure is Nm^{-2} of pascal where 1 Pa = 1 Nm^{-2}.

Summary of some laws of pressure in fluids at rest

1. The pressure at a depth h in a fluid at **rest**, **due to the fluid itself**, is $P = h\rho g$ pascals where ρ is the density of the fluid.



$$Weight\ of\ water\ in\ a\ cylinder = mg$$
$$but\ m = V\rho, then$$
$$Weight\ of\ water\ in\ a\ cylinder = V\rho g$$
$$and\ V = Ah, so$$
$$Weight\ of\ water\ in\ a\ cylinder = Ah\rho g$$

Substituting in the equation of pressure

$$P = \frac{F}{A} = \frac{Ah\rho g}{A}$$

$$\boxed{P = h\rho g}$$

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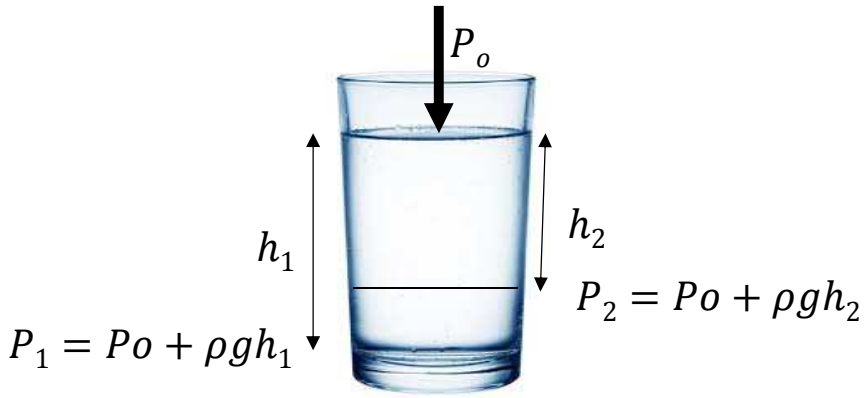
2. At any point in a liquid which is at rest the **total pressure** is the pressure on the **surface** of the liquid **plus the pressure due to the liquid itself**



3. At any points in the **same horizontal plane** in any one liquid which is at rest, the pressures are the **same**. (Otherwise the liquid would flow.)

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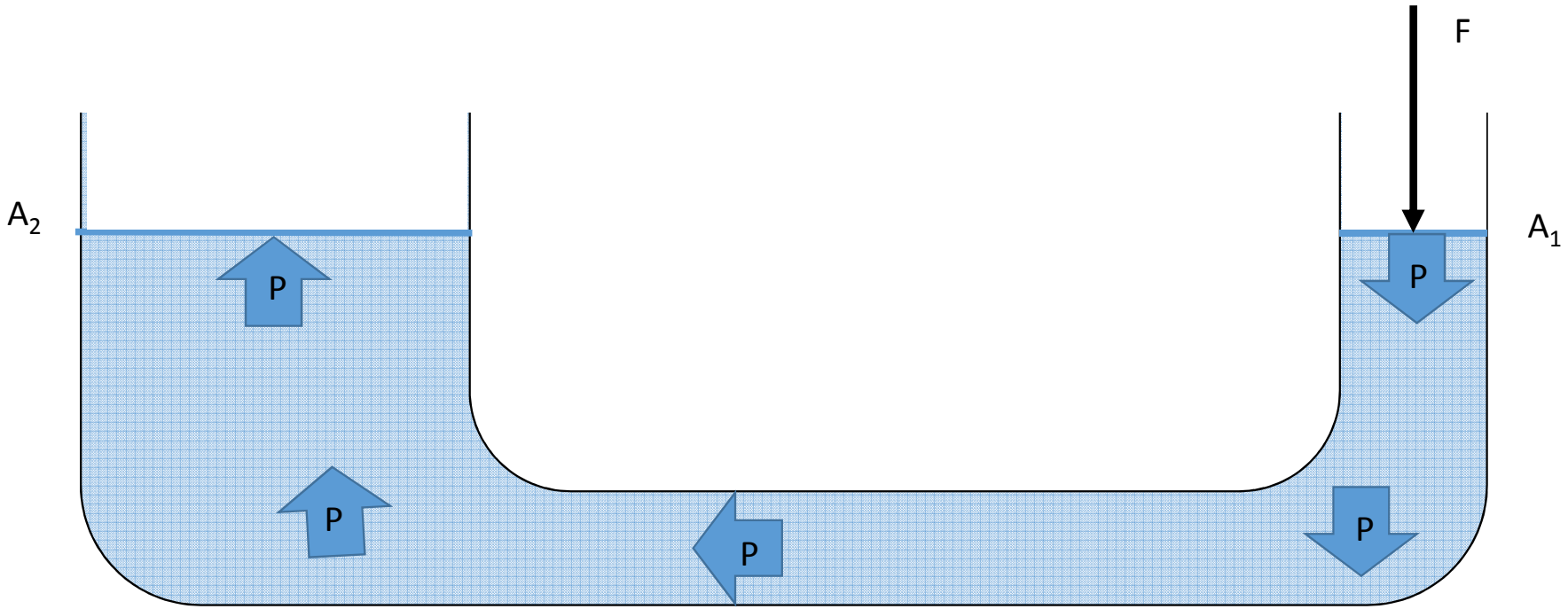
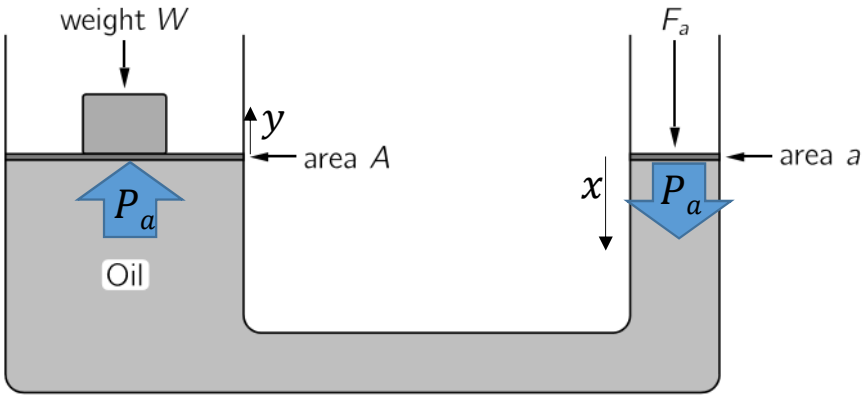
4. Pressure applied to the surface of a liquid is transmitted equally throughout the liquid in every direction. (This is called Pascal's Law.) This principle is used in the Hydraulic Press



The hydraulic press



Consider applying a force F_a on the left cylinder in the diagram, over the area a, in an attempt to lift an object of weight W. The pressure exerted on this cylinder is given by

$$P_a = \frac{F_a}{a}$$

By Pascal's law, the P_a exerted over the area a is equal the pressure exerted over area A. The Force on area A is given by

$$F_A = PA$$
$$F_A = PaA$$
$$F_A = \frac{F_a}{a}A$$

If A = 1.0 m^2 and a = 0.1 m^2, then $F_A = Fa\frac{1}{0.01} = 100Fa$

$$F_A = F_a\frac{A}{a}$$

If F_a forces the liquid in the right cylinder to move down by x m, by how much does the liquid in the other cylinder move up? Let say it moves y m. The change in volume in both cylinders is the same,

$$Ay = ax \text{ ,so}$$

$$y = \frac{a}{A}x$$

If A = 1.0 m^2, a = 0.1 m^2 and $x = 0.05\ m$, then $y = 0.005\ m$

Tutorial problems

I2 A water bed has dimensions 1.8m ×2.1m ×0.25m. The floor of the bedroom will tolerate an additional load of no more than 7000 N. Determine whether the bed should be purchased.

I3 Numerous jewellery items of silver are melted down and cast into a solid circular disk that is 0.0200 m thick. The total mass of the jewellery is 10.0 kg. Find the radius of the disk. (The density of silver is 1.05×10^4 kgm^{-3} .)