Surname & initials:

University of KwaZulu-Natal

Electromagnetism Test2 (2016) : Phys365

Time: 90 minutes **Total marks:** 90

General Instructions

- ☞ Check that you have two pages.
- ☞ Answer ALL questions.
- ☞ A detailed explanation of your steps and the physics involved must accompany any mathematics.
- ☞ Use proper vector and tensor notation throughout.
- ☞ Candidates are reminded to write clearly and legibly.

Question 1

(a) (i) Write down Laplace's equation. What name is given to its solutions? (3)

(ii) Suppose V_1 and V_2 are both solutions to Laplace's equation. Prove that $V = \alpha_1 V_1 + \alpha_2 V_2$ is also a solution (here the α_i are constants). (3)

(b) Use Cartesian tensors to derive:

(i) the $\mathbf{E}$ field corresponding to the electrostatic potential $V(\mathbf{r}) = V_0 - \mathbf{E}_0 \cdot \mathbf{r}$ where V_0 and $\mathbf{E}_0$ are constants. (3)

(ii) the $\mathbf{B}$ field corresponding to the magnetic vector potential $\mathbf{A}(\mathbf{r}) = \dfrac{\mu_0}{4\pi}\dfrac{\mathbf{m}\times\mathbf{r}}{r^3}$. (15)

[24]

Question 2

(a) Write down the integral form of Gauss's law in *magnetism*. Now express this law in differential form. (4)

(b) (i) Prove that the equation $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ (here $\mathbf{J}$ is the current density) violates a fundamental law of physics. (2)

(ii) Give the name of the law which is violated. (2)

(iii) Hence modify the equation in (i) to obtain one of Maxwell's vacuum equations. (8)

[16]

Question 3

A plane, monochromatic electromagnetic wave (angular frequency ω; wavevector $\mathbf{k}$) propagates in vacuum where there are no free charges or currents. Prove that the wave is transverse. You may assume that $\nabla e^{i\mathbf{k}\cdot\mathbf{r}} = i\mathbf{k}e^{i\mathbf{k}\cdot\mathbf{r}}$. (15)

[15]

PTO

Question 4

The electric field of a plane electromagnetic wave propagating in vacuum is

$$E_x = 0, \qquad E_y = 0, \qquad E_z = E_0 \cos(ky + \omega t).$$

(i) Use a Maxwell equation to derive the magnetic field components of this wave. (15)

(ii) Hence calculate the time average Poynting vector $\langle \mathbf{S} \rangle$. (2)

[17]

Question 5

In this Question the electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ transform from a frame S to a frame S′ according to:

$$\left.\begin{array}{lll} E'_x = E_x & E'_y = \gamma(E_y - vB_z) & E'_z = \gamma(E_z + vB_y) \\ B'_x = B_x & B'_y = \gamma(B_y + \frac{v}{c^2}E_z) & B'_z = \gamma(B_z - \frac{v}{c^2}E_y) \end{array}\right\},$$

where all symbols have their usual meaning.

(a) Prove that $\mathbf{E} \cdot \mathbf{B}$ is invariant. (9)

(b) Suppose that $\mathbf{E} = 6 \times 10^6 \hat{\mathbf{z}}\,\mathrm{Vm}^{-1}$ and $\mathbf{B} = 0$. Calculate $\mathbf{E}'$ and $\mathbf{B}'$ in a frame S′ moving with velocity $\mathbf{v} = \frac{1}{2}c\hat{\mathbf{x}}$ relative to S. (9)

[18]