UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

School of Chemistry & Physics
Pietermaritzburg Campus
November 2013: Main Examination
Physics 362: Statistical Physics & Condensed Matter

TIME: 3 hours	**MAXIMUM MARKS:** 180

Internal Examiner(s):	**External Examiner:**
Dr G Pellicane & Prof. G Mola	Dr D Naidoo (University of the Witwatersrand)

General instructions

- There are two sections in this examination. Section A: **Statistical Physics** & Section B: **Condensed Matter**. Please answer each Section in separate examination answer booklets.
- Make sure that you have a total of six sets of questions which are composed of three sets of questions from each section. Each question is equal to 30 Marks
- Answer all questions. Write your solutions as clearly as possible.
- You are allowed to use only **one and half hours** for each section.
- All symbols used retain their usual meaning in Physics.

STATISTICAL PHYSICS

$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{N,V}$	$\binom{N}{N_1} = \frac{N!}{N_1!(N-N_1)!}$	$\frac{P}{T} = \left(\frac{\partial S}{\partial V}\right)_{N,E}$
$\bar{N}_1 = pN$	$C_V = \left(\frac{\partial \bar{E}}{\partial T}\right)_{N,V}$	$F = -k_B T lnZ$
$S = k_B(lnZ + \beta\bar{E})$	$\zeta = (\frac{2\pi m k_B T}{h^2})^{\frac{3}{2}}$	$P_N = \bar{N}^N \frac{e^{-\bar{N}}}{N!}$

Answer all 3 questions, comprising of 30 marks each.
All symbols have the usual meaning.

Question 1.

A metal is evaporated in vacuum from a hot filament. The resultant metal atoms are incident upon a quartz plate some distance away and form a thin metallic film. The metal atoms can be assumed equally like to impinge upon any element of area of the plate.

(i) If one considers an element of substrate area of size σ^2 (where σ is the metal atom diameter), show that the number of metal atoms piled up on this area should be distributed according to a Poisson distribution. (11)

(ii) Suppose that one evaporates enough metal to form a film of mean thickness corresponding to 6 atomic layers. (a) What fraction of the substrate area is covered by metal layers 3 and 6 atoms thick, respectively? (b) What fraction is not covered by metal at all? (5)

Question 1(iii) continues over the page .....

(iii) Consider the average thickness of metal on the substrate resulting from evaporating 2 g of Aluminium (atomic mass 27, $\sigma = 0.286$ nm) on an area $A = 0.0818$ m^2 ($N_A = 6.022 \times 10^{23}$). (a) Calculate the fraction of the substrate that is not covered by any metal at all in this case. (b) How many grams of Aluminium should be evaporated in order to obtain the same fraction of the substrate free from metal as in the case before? (14)

[30]

Question 2

A system consists of N impurity atoms in a solid matrix. The microstates of the system are specified by the set of occupation numbers n_i, in which $n_i = 0$ or 1 depending on whether the i^{th} impurity is in its ground or excited state. The overall energy is

$$E = \epsilon \sum_{i=1}^{N} n_i = \epsilon N_1 \, ,$$

in which $N_1 = E/\epsilon$ is the total number of excited impurities. The number of microstates $\Omega(E)$ with energy between E and $E + \delta E$ is given by

$$\Omega(E) = \binom{N}{N_1} \frac{\delta E}{\epsilon}$$

(i) Show that the entropy of the system can be written as (neglecting constant terms):

$$S = -Nk_B \left[\frac{E}{N\epsilon} ln \frac{E}{N\epsilon} + \left(1 - \frac{E}{N\epsilon}\right) ln \left(1 - \frac{E}{N\epsilon}\right) \right].$$

(10)

(ii) (a) Determine the energy of the system as a function of the temperature T, and show the range of energies in which T is positive. (b) Give an explanation why this model exhibits a range of negative temperatures. (10)

(iii) (a) Find the heat capacity C of the system and show explicitly that it vanishes at both high and low temperatures. (b) Provide a physical interpretation of this result. (10)

[30]

Question 3.

The entropy of the ideal, monoatomic gas can be written as

$$S = NS_0 + Nk_B \ln \left[\left(\frac{E}{E_0}\right)^{\frac{3}{2}} \left(\frac{V}{V_0}\right) \left(\frac{N}{N_0}\right)^{-\frac{5}{2}} \right].$$

in which E is the energy, V is the volume, and N is the total number of particles.

(i) (a) Find any two equations of state and show explicitly that this expression of the entropy is not affected by the Gibbs paradox. (11)

Question 3(ii) continues over the page .....

(ii) Determine the Helmholtz free energy by exploiting the proper Legendre transform, and show that

$$\left(\frac{\partial F}{\partial T}\right)_{N,V} = -S.$$

(7)

(iii) Given the partition function of the ideal gas:

$$Z = \frac{V^N \zeta^N}{N!},$$

(a) determine the Helmholtz free energy and the entropy using the Stirling formula ($ln(N!) = Nln(N) - N$), hence (b) verify that their dependence on the thermodynamic parameters is equivalent to the expressions obtained/reported previously.

(12)

[30]

Total Marks [90]

SECTION B
CONDENSED MATTER

PHYSICAL CONSTANTS AND MATHEMATICAL FORMULAS

Planck's constant:	$h = 6.626 \times 10^{-34}\, J \cdot s = 4.136 \times 10^{-15}\, eV \cdot s$
Electron rest mass:	$m_e = 9.11 \times 10^{-31}$ kg (rest mass $m_e = 0.511$ MeV/c^2)
Proton rest mass:	$m_p = 1.6726 \times 10^{-27}$ kg (rest mass $m_P = 938.25$ MeV/c^2)
Electron Charge:	$e = 1.6 \times 10^{-19}$ C
Speed of light:	$c = 3.0 \times 10^8$ m/s
	hc = 1240 eV·nm
Boltzmann constant:	$k_B = 1.3806 \times 10^{-23}$ J·K^{-1} ($k_B = 1.3806 \times 10^{-16}$ erg·K^{-1})
Avogadro's number:	$N_a = 6.023 \times 10^{23}$/mole

Taylor series of a function $f(x)$ around $x = a$

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$$

Important examples of Taylor series for small x:

$$e^x \approx 1 + x + \cdots \quad ; \quad \sqrt{1+x} \approx 1 + \frac{1}{2}x + \cdots$$

Integrals: $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}} \quad ; \quad \int_0^{\infty} \frac{x^4 e^x}{(e^x - 1)^2} dx = \frac{4}{15}\pi^4$

Condensed Matter Questions: 4, 5 & 6.

Question 4.

(Crystal structure)

(1) Solid can be classified as single crystal, polycrystal and amorphous based on the arrangements of atoms in space. Briefly, discuss each class of the solid. (6)

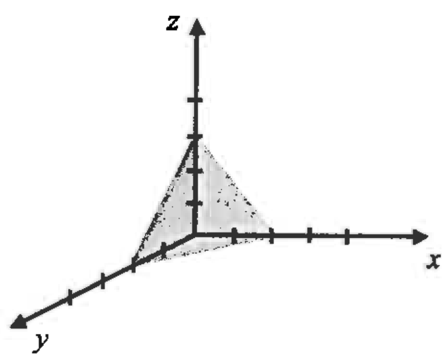
(2) Write down at least four main symmetry operations in a crystal structure. (2)

(3) Construct a Wigner-Seitz primitive cell in two dimensional hexagonal lattice. (3)

(4) In a monoatomic solid, atoms can be viewed as a packing of solid spheres. List the possible close packing of the spheres in two dimensions. (2)

(5) Calculate the lattice constant for rock salt of density 2180 kg/m^3 assuming that it has fcc structure. Molecular weight of NaCl is 58.5. (5)

(6) A crystal plane intercepts the reference axes at 0.5, 1 and 2 (not necessarily the same as in the figure). Find the Miller indices of the plane. (2)



(7) What does a Brillouin zone represent in the study of crystal structure? (4)

(8) A beam of electrons with kinetic energy 2 keV is diffracted as it passes through a polycrystalline metal foil. The metal has a cubic crystal structure with a spacing $1.2\mathring{A}$.

(i) Calculate the wavelength of the electron. (3)

(ii) Calculate the Bragg angle for the second order diffraction maximum. (3)

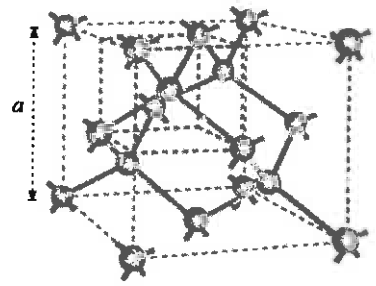
(9) Discuss the properties of a phonon in condensed matter. (3)

[30]

Question 5.

(Atomic packing and reciprocal lattice)

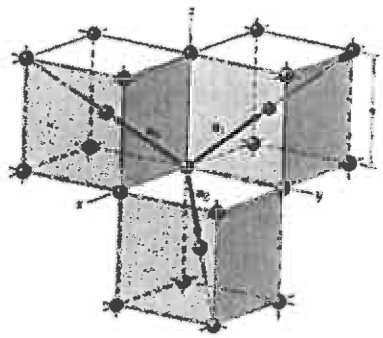
(a) The diamond structure shown below is one of the cubic families in a lattice.



(i) Determine the number of atoms in a diamond unit cell based on the structure given above. Is it a primitive unit cell? If not, why? (5)

(ii) Calculate the packing efficiency of a diamond structure (bonding angle 109.5°). (8)

(b) The primitive translation vectors of the body-centred cubic lattice (bcc) are given as shown in the figure below.



These vectors connect the lattice point at the origin to lattice points at the body centres. In terms of the cube edge a of the bcc, the primitive translation vectors are:

$$\vec{a}_1 = \tfrac{1}{2}a(-\hat{x}+\hat{y}+\hat{z}); \qquad \vec{a}_2 = \tfrac{1}{2}a(\hat{x}-\hat{y}+\hat{z}); \qquad \vec{a}_3 = \tfrac{1}{2}a(\hat{x}+\hat{y}-\hat{z})$$

Question 5(b)(i) continues over the page ...

(i) Calculate the volume of the primitive cell. (4)

(ii) Determine the corresponding primitive translation vectors in the reciprocal lattice space. (7)

(iii) From (ii) what kind of structure can you get in reciprocal lattice space? (2)

(iv) Calculate the nearest neighbour distance. (4)

[30]

Question 6.

(Thermal properties of solid)

(a) Derive the dispersion relation for the lattice vibrations of a chain of identical masses M, in which each is connected to its nearest neighbours by springs of spring constants C. Take an equilibrium spacing between atoms as 'a'. (8)

(b) The specific heat of solid is known to depend on temperature but the reason for such temperature dependence was not fully understood until the birth of quantum mechanics. It was Einstein who first proposed a model that paved the way for understanding the physics behind the dependence of specific heat on temperature.

(i) State the key assumptions in Einstein theory of heat capacity. (3)

(ii) Find the specific heat of solid at constant volume based on the Einstein model. (7)

(iii) Discuss the limiting case scenarios; i.e $kT >> \hbar\omega$ and $kT << \hbar\omega$. (4)

(iv) Draw the general trend of the specific heat of solid as a function of temperature. (3)

(c) Draw the energy band diagram of a semiconductor. Discuss why semiconductors differ from insulators in terms of the energy band gap. (5)

[30]

TOTAL MARKS [90]