UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

School of Chemistry & Physics
Pietermaritzburg Campus
November 2014 : Main Examination
Physics 362: Statistical Physics & Condensed Matter

TIME: 3 hours	**MAXIMUM MARKS:** 180

Internal Examiner(s):	**External Examiner or Moderating Board:**
Dr G Pellicane and Prof G Mola	Prof D Naidoo (UNIVERSITY OF THE WITWATERSRAND)

GENERAL INSTRUCTIONS:

1. Check that you have the correct number of pages.
2. Candidates are reminded to be as thorough as possible and to write legibly.
3. Please begin your answers on a new page for each problem in the answer booklets.

STATISTICAL PHYSICS

$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{N,V}$	$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$	$Z = \sum_r e^{-\beta E_r}$
$\bar{N}_1 = pN$	$\bar{E} = \bar{M} \cdot H$	$\bar{E} = -\frac{\partial \ln Z}{\partial \beta}$
$\bar{n} = \eta \frac{\partial \ln Z}{\partial \eta}$	$P_{N_1} = \frac{N!}{N_1!(N-N_1)!} p^{N_1} q^{N-N_1}$	$P_N = \bar{N}^N \frac{e^{-\bar{N}}}{N!}$

Statistical Physics
Answer all 3 questions, comprising of 30 marks each.
All symbols have the usual meaning.

Question 1

DNA unfolds during the transcription process when it makes messenger RNA. Consider DNA to have the structure of a horizontal ladder with some of the rungs (links) broken (open). A link n can only open if all of the links to the left of it $1, 2, ..., n-1$ are open. Suppose DNA has N links, and each of them can be in one of the two energy states 0 (closed) or ϵ (open).

(i) Calculate the partition function of the system. (10)

(ii) Calculate the average energy $\overline{E}$ of open links, and write an expression for the low-temperature limit of it. Provide also your comments about the associate physical behaviour in that limit. (10)

(iii) Calculate the average number $\overline{n}$ of open links and verify that

$$\overline{n} = \frac{\overline{E}}{\epsilon}.$$

(10)

[30]

Question 2.

Consider an isolated system of a large number of very weakly interacting localized particles with spin 1/2. Each particle has a magnetic moment μ which can point either parallel or anti-parallel to an applied uniform magnetic field H. The energy of the system is then $E = -(N_1 - N_2)\mu H$, where N_1 is the number of spins aligned parallel to H, and N_2 is the number of spins aligned anti-parallel, and $N = N_1 + N_2$.

(i) Show that the number of states $\Omega(E)$ with energy between E and $E + \delta E$ is

$$\Omega(E) = \frac{N!}{(\frac{N}{2} + \frac{E}{2\mu H})!(\frac{N}{2} - \frac{E}{2\mu H})!} \times \frac{\delta E}{2\mu H}.$$

(11)

(ii) Assuming that the logarithm of $\Omega(E)$ is written as

$$ln\Omega(E) = ln\frac{\delta E}{2\mu H} + Nln(N) +$$

$$-\left(\frac{N}{2} - \frac{E}{2\mu H}\right) ln\left(\frac{N}{2} - \frac{E}{2\mu H}\right) - \left(\frac{N}{2} + \frac{E}{2\mu H}\right) ln\left(\frac{N}{2} + \frac{E}{2\mu H}\right),$$

find under what circumstances the temperature is defined positive, and explain why this model exhibits a range of negative temperatures. (8)

(iii) Express the total magnetic moment of the system as a function of temperature, and show that in the limit of high-field, low-temperature the resulting expression gives rise to magnetic saturation.

(11)

[30]

Question 3.

A cylinder is segmented by an adiabatic and impermeable wall, dividing its volume in the ratio 2 : 1. The bigger section of the box contains 900 ideal-gas molecules of the chemical species A, and the smaller one 300 ideal-gas molecules of the chemical species B. A small hole is made on the wall, and you wait for thermodynamic equilibrium to be established inside the cylinder.

(i) Provide a comprehensive explanation of the reason why the probability to find particles of any species in one of the two parts of the box is given by the binomial distribution. (9)

(ii) Calculate the average number of molecules of each kind in the two sections. (5)

(iii) Calculate the probability that the system could be found again in the initial state - 900 gas molecules of the chemical species A in the bigger section, and 300 molecules of the chemical species B in the smaller section. Clearly justify any formula and numerical procedure adopted. (8)

(iv) Assume that the volume of the smaller part is equal to $\frac{1}{10^6}$ of the one of the bigger part of the box. What are the probabilities of finding one particle of each species in the smaller part of the box? Provide a clear justification for the adopted procedure to calculate the probabilities. (8)

[30]

Total Marks [90]

Condensed Matter
Answer all 3 questions.
All symbols have the usual meaning.

QUESTION 1

(Crystal structure)

(1) How many Bravais lattices are there in two dimensions? Write down the names. (3)

(2) Write down at least four main symmetry operations in a crystal structure. (2)

(3) Construct a Wigner-Seitz primitive cell in two dimensional hexagonal lattice. (3)

(4) A crystal has face centred cubic (FCC) structure in direct lattice. What kind of structure would the crystal be in reciprocal lattice? (2)

(5) Silicon has diamond type structure and density $2329\,\mathrm{kg/m^3}$. Calculate the dimensions of the unit cell and the atomic diameter. (use atomic weight of Si Wt = 28.08,) (6)

(6) Determine the Miller indexes of the crystal planes that intercept the reference axes at

(i) -0.25, 0.5, -1

(ii) 2, 3, 1 (3]

(7) A beam of electrons with kinetic energy 2 keV is diffracted as it passes through a polycrystalline metal foil. The metal has a cubic crystal structure with a lattice spacing $1.2\mathring{A}$.

(i) Calculate the wavelength of the electron. (3)

(ii) Calculate the Bragg angle for the second order diffraction maximum. (3)

(8) Plot and discuss the properties of the total interaction potential between atoms in ionic solid. (4)

(9) The dispersion relation of an acoustic wave in solid can be expressed as $\omega = C_0 \sin(\frac{1}{2} Ka)$ where C_0 and a are constants. Calculate the group velocity. (3)

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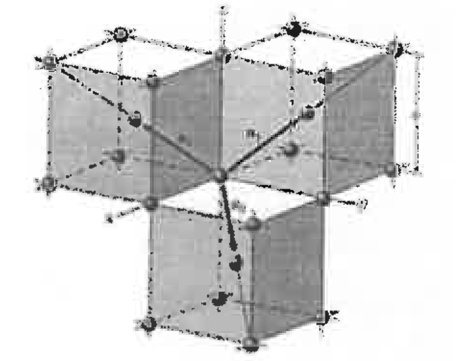
QUESTION 2

(Reciprocal lattice and Lattice Vibration)

(1) The primitive translation vectors of the body-centred cubic lattice (bcc) are given as shown in the figure below.



These vectors connect the lattice point at the origin to lattice points at the body centres. In terms of the cube edge **a** of the bcc, the primitive translation vectors are:

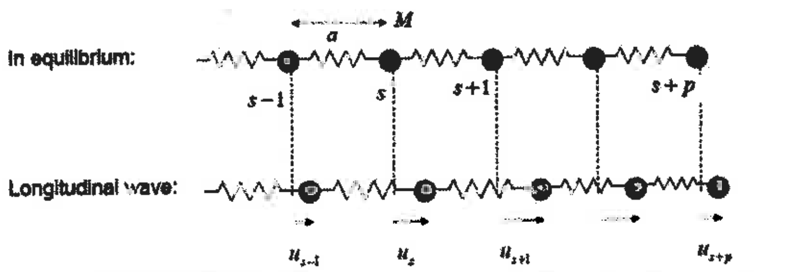
(i) Calculate the volume of the primitive cell. (4)

(ii) Determine the corresponding primitive translation vectors in the reciprocal lattice space. (7)

(iii) From (ii) what kind of structure can you get in reciprocal lattice space? (2)

(iv) Calculate the nearest neighbour distance. (4)

(2) Consider a monoatomic solid subject to lattice vibrations. Each atom has mass M and arranged in a linear chain like structure with lattice constant "**a**". Atomic oscillations in solid can be approximated by a harmonic oscillator with spring constants C. Taking into account only the nearest neighbor interactions with reference atom derive the dispersion relation and discuss the result.



(11)

[28]

QUESTION 3

(Thermal properties of solid)

(1) Acoustic properties of dielectric solids dominate their thermodynamic behavior and other properties such as photoconducting resistance. Diamond is a monatomic dielectric solid of carbon having 10^{21} atoms/cm^3.

- i) Sketch the specific heat of solid as a function of absolute temperature and discuss the general behavior for low and high temperature ranges. (3)
- ii) Using Debye's descriptions of state per frequency range calculate the total number of modes N . (5)
- iii) If the acoustic velocity at low frequency is 5×10^5 cm/sec, what will be the approximate value of ω_D? (5)
- iv) Find the wave vector at the cut off frequency (i.e. K_D) (3)
- v) Find is the Debye's temperature at cut off frequency? (4)

(2) Qualitatively discuss the origin of the energy band gap in solid and plot the energy band diagram. (5)

(3) What are the roles of intentionally doped impurities in semiconductor? Show the position of impurity levels in the energy band diagram. (5)

[30]

Total Marks [90]

PHYSICAL CONSTANTS AND MATHEMATICAL FORMULAS

Planck's constant:	$h = 6.626 \times 10^{-34}\, J \cdot s = 4.136 \times 10^{-15}$ eV·s
Electron rest mass:	$m_e = 9.11 \times 10^{-31}$ kg (rest mass $m_e = 0.511$ MeV/c^2)
Proton rest mass:	$m_p = 1.6726 \times 10^{-27}$ kg (rest mass $m_P = 938.25$ MeV/c^2)
Electron Charge:	$e = 1.6 \times 10^{-19}$ C
Speed of light:	$c = 3.0 \times 10^{8}$ m/s
	hc = 1240 eV·nm
Boltzmann constant:	$k_B = 1.3806 \times 10^{-23}$ J·K^{-1} ($k_B = 1.3806 \times 10^{-16}$ erg·K^{-1})
Avogadro's number:	$N_a = 6.023 \times 10^{23}$ atoms/mole

Taylor series of a function $f(x)$ around $x = a$

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots$$

Important expansion series: for small x: $e^x \approx 1 + x + \cdots$; $\sqrt{1+x} \approx 1 + \frac{1}{2}x + \cdots$

Integrals:

$$\int_{-\infty}^{\infty} \frac{x^2 e^x}{(e^x+1)^2} dx = \frac{\pi^2}{3} \quad ; \quad \int_0^{\infty} \frac{x^4 e^x}{(e^x+1)^2} dx = \frac{4\pi^4}{15}$$

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}} \quad ; \quad \int_0^{\infty} x^{\frac{1}{2}} e^{-x} dx = \frac{\sqrt{\pi}}{2}$$

$$f(\varepsilon, T) = \frac{1}{\exp[(\varepsilon - \mu)/k_B T] + 1}$$