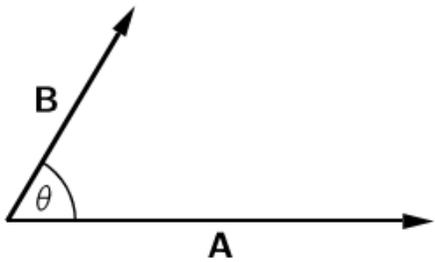# Addition of vectors using parallelogram
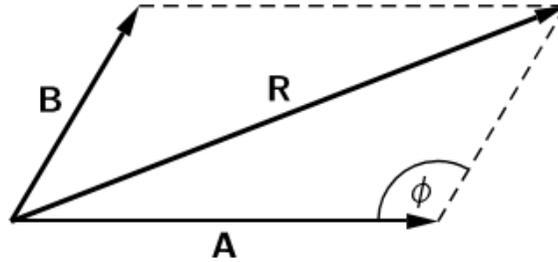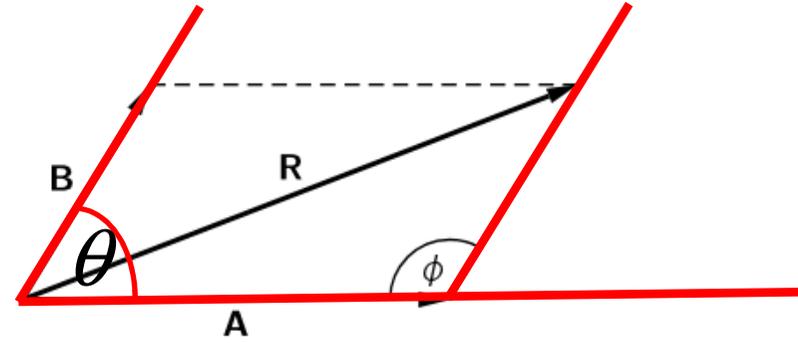

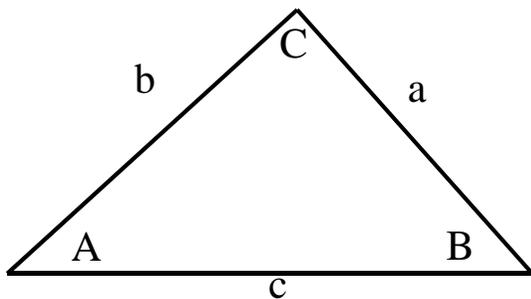
(a)   (b)   (c)

Consider two vectors **A** and **B** at an angle θ to each other as in Figure 1.5a. If we construct lines parallel and equal to A and B, we obtain a parallelogram with the resultant R as the diagonal (Figure b). Using the fact that the sum of co-interior angle equal to 180^0, then $\phi = 180 - θ$ (see Figure (c)).
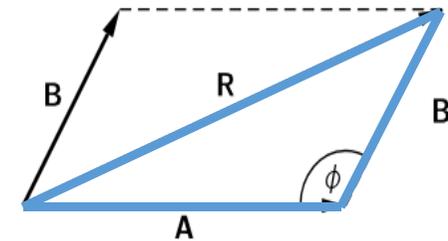

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Knowing the magnitudes of **A**, **B** and ϕ, **R** can be determined using the **cosine rule**:

$$R^2 = A^2 + B^2 - 2AB \cos \phi$$

Example 1.6: now using the parallelogram method

Suppose a car travels from point A in an easterly direction for 10 km to point B, and then travels another 5 km in a direction 60^0 north of east to point C. Determine the total (i) **distance** travelled and the **displacement** of the car using a parallelogram method.

You should be able to do tutorials A1 to A10

A1 (a) $5 \times 10^{-3} \, g$, (b) $5 \, mg$, (c) $5 \times 10^3 \, \mu g$

A2 (a) $96 \, km \, h^{-1}$ (b) $26.7 \, m \, s^{-1}$

A3 One hectare is the area of a square of sides 100 m. How many square metres is this? If a 2 hectare rectangular property has a width of 20 m, what is the length of the property?

A4 If there are 2.47 acres in one hectare. How many square metres is one acre? (See Question A3 for the size of a hectare.)
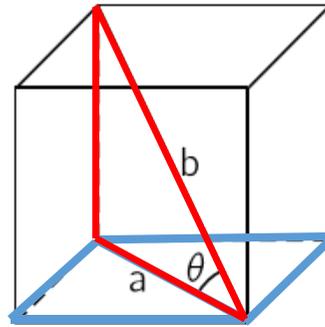
A5 Express the volume of one litre in cubic metres given that one millilitre equals one cubic 3 centimetre (1ml = 1 cm).
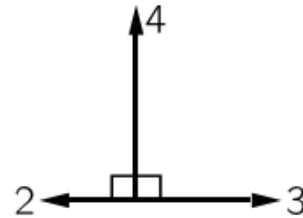
A6 35.5°

A7 $y = 0.26 \, km, \quad x = 0.97 \, km$

A8 The diagram alongside shows a cube with sides of length 10 cm. Determine the lengths of the diagonals a and b and the angle θ.

A9 Vectors of magnitude 3, 4 and 2 units are oriented as shown. Find their resultant in magnitude and direction (a) by drawing and (b) by calculation.

A10 A ship is moving in a sea in which there is a W to E current of 5 ms^{-1} . If the ship is steering N by its own compass, and is moving through the water at 15 ms^{-1}, with what speed and in what direction is it moving relative to the earth's surface?

1. An object moves 7 m to the East, 3 m to the South, then 11 m West. Find the magnitude of its total displacement.

2. Two horses are pulling a cart. One horse pulls with a force of 50 N towards the east and another horse pulls with a force 30 N at 30^0 north of east.
(a) Draw a diagram to show the direction of the resultant
(b) Find the magnitude and the direction of the resultant using addition of vectors using components
(c) Find the magnitude of the resultant using parallelogram method

Kinematics in one dimension

To set an object in motion, a force has to be exerted on it. **Kinematics** is the study of objects which are already in motion, disregarding the force that caused the motion in the first place. A study of the forces will be considered in Chapter 3. To describe the motion of an object, we need to specify the position of the object with respect to some **frame of reference** and a **reference point**. A frame of reference is the set of coordinate system used to measure the properties of an object's motion such as displacement, velocity, acceleration etc. A reference point is the point where we start measuring these properties. The Cartesian coordinate system may be considered as a frame of reference where its origin is a reference point and the x-, y-, and z-axes are the directions the object may take. In this section we will consider the motion of objects in one direction (i.e. one dimension motion). The properties of an object that we will consider are displacement, speed, velocity and acceleration.
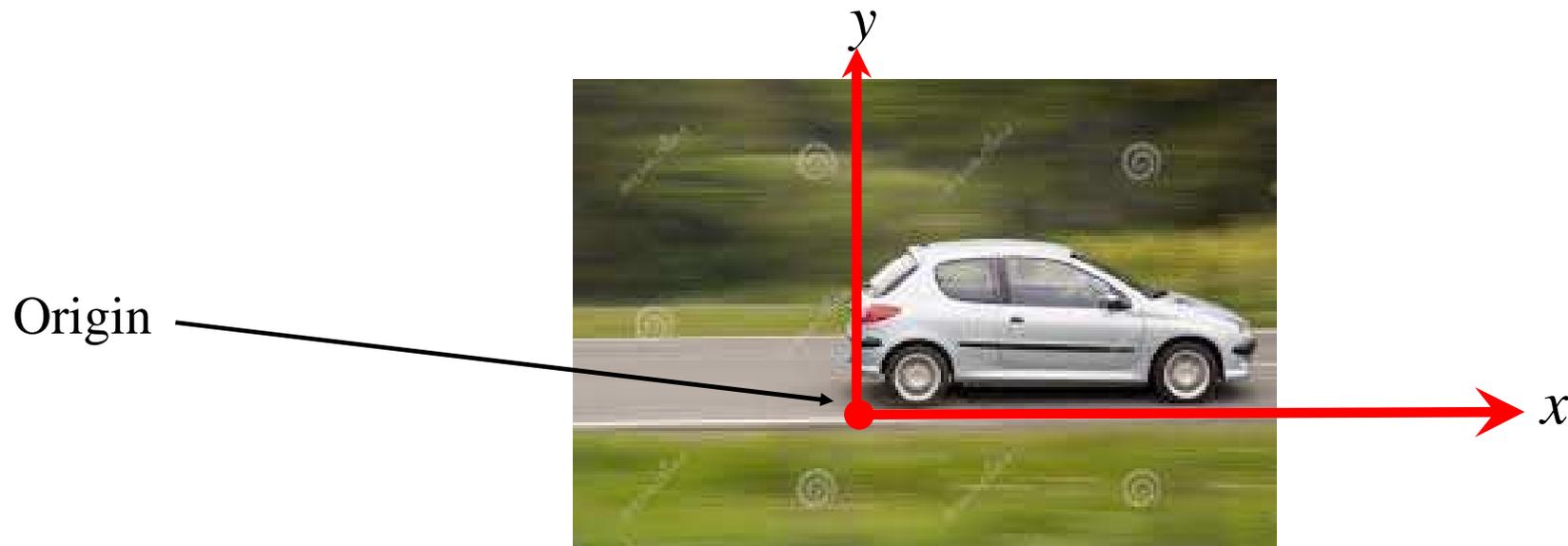
Displacement

Displacement is the change in position of an object in a specified time interval. Displacement can also be understood as a distance from the initial position to the final position along the **straight line**.
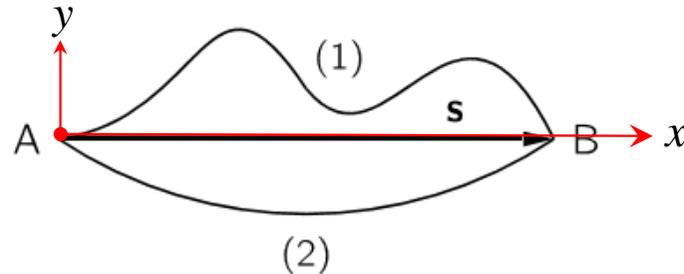
Consider two people travelling from point A to point B through path (1) and (2). The person travelling through path (1) will travel a longer **distance** than the person travelling through path (2). However, their **displacement** is the same.
If we align the x-axis of our coordinate system along the direction of the displacement, we can determine the magnitude of the displacement using

$$s_x = \Delta x = x_f - x_i$$

where x_f and x_i are final and initial positions and Δ is delta which indicates the change.

Average Speed

The speed of an object is a measure of how fast it is moving. Average speed is the total distance travelled divided by the time taken, mathematically it is given by:

$$Average\ speed\ = \frac{Total\ distanve}{Elapsed\ time}$$

If an object coves the same distance after a given equal time, that object is said to at **uniform speed**. In this case the graph of the distance travelled as a function of time taken yield a straight line (left graph below) and the slope of the graph is equal to the speed. If the distance travelled after a given time keeps on increasing, the object's speed is said to be non-uniform. The graph of the distance travelled against elapsed time is a parabola (right graph below).

Uniform Velocity — Distance vs Time

Non-uniform Velocity — Distance vs Time
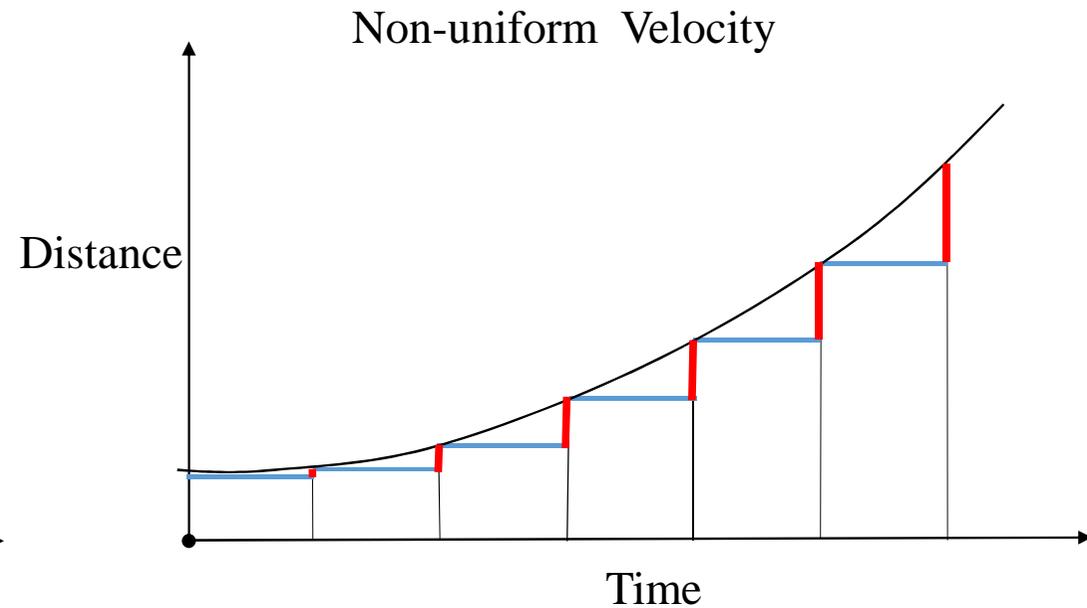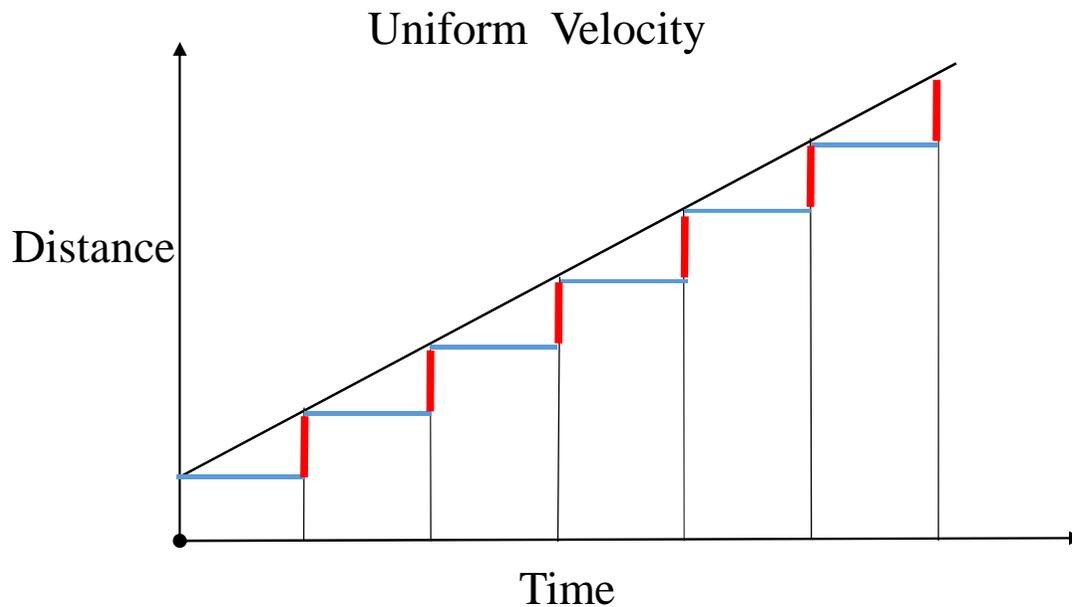
Average Velocity

The average velocity in a given **direction** is the total displacement in that direction divided by the total time taken, Mathematically it is given by:

$$\bar{v} = \frac{s}{\Delta t}$$

In the equation above $\Delta t = t_f - t_i$ is the total time and the bar indicate it is an average velocity. For a motion in a straight line along the x-axis, the only non-zero component in the x-direction is v_x. Thus for convenience we will omit the subscript x and write:

$$\bar{v} = \frac{\Delta x}{\Delta t} \qquad \text{where } \Delta x = x_f - x_i$$

Instantaneous Velocity

The instantaneous velocity is the velocity at any instant during the motion of an object. In order to obtain an estimate of the instantaneous velocity, we measure the displacement over a very small time interval. As the time interval of our measurement gets smaller, the average velocity will approach the instantaneous velocity. In what follows we will refer to the instantaneous velocity simply as the velocity.

Example 2.1: Velocity and speed

A jogger goes for her usual afternoon run. She leaves from her house and jogs a round trip of 10 km in 1 hour. What is her average speed and her average velocity after 1 hour?

Example 2.2: Position-time graph

A cyclist rides at constant velocity and then stops for lunch. After lunch, she rides back to the place from where she started, at a different constant velocity.

(a) Find the average velocity in each of the regions A, B and C indicated on the graph below
(b) Find the total distance travelled by the cyclist
(c) Find the displacement cover by the cyclist
(d) Find the average speed for the whole journey
(e) Find the average velocity for the whole journey