Application Of Ampère's Law And Magnetic Domains

Dr Miguel Cavero

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Ampère's Law

The integral form of Ampère's Law is

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

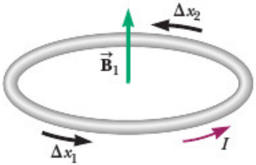
where the integral is over a *closed* path which surrounds the current I. The dot product $\mathbf{B} \cdot d\mathbf{l}$ can be written as $Bdl \cos\theta$, where θ is the angle between **B** and $d\mathbf{l}$, i.e. the product of the parallel component of **B** and the line segment $d\mathbf{l}$.

Magnetic Field Of Current Loops

Current in a conductor (such as a wire) produces a magnetic field, where the field lines are circles around the wire (for a long, straight wire).
The strength of the magnetic field depends on the amount of current in the conductor.
The strength of the magnetic field can also be enhanced by shaping the conductor into a loop.
For a conductor shaped into a loop that carries a current - i.e. a current loop - the second right-hand rule shows the direction of the magnetic field is into/out of the plane of the loop.
The direction of the magnetic field at the centre of a (symmetrical) current loop is perpendicular to the plane of the loop.

Magnetic Field Of Current Loops

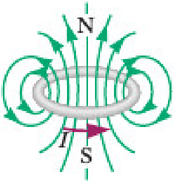
Consider a current loop, where the current is moving in an anti-clockwise direction. For two small segments within the loop, the second right-hand rule shows that the magnetic field at the centre of the loop points upwards.



For *every* small segment, the magnetic field points upwards *inside* the loop. (What about the direction of the field outside of the loop?)

Magnetic Field Of Current Loops

The magnetic field lines around for all regions inside the loop may be represented as follows:



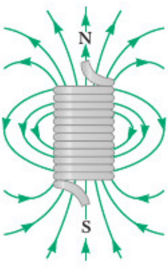
Note that the field line pattern for this current loop is similar to the pattern of a bar magnet, with one side of the current loop resembling the north pole of a magnetic while other side resembles a south pole.

Magnetic Field Of A Solenoid

A solenoid is a long wire bent into a coild many closely spaced loops. It is sometimes referred to an electromagnet - it acts as a magnet only when there is a current in the solenoid.
(Solenoids are similar to, but not the same as, inductors. Inductors generally are used to control current flow and store energy. Solenoids are usually used to provide a magnetic field.)
The magnetic field inside the solenoid is directly proportional to the current. It is also proportional to the number of coils (or turns) per unit length.
Consider a long solenoid (ideally of infinite length) with many turns that are closely spaced.
For a very long solenoid, the field inside can be taken as uniform. For a solenoid whose length is much greater than its diameter, the field lines leaving the north pole end spread out over a wide region before returning to the south pole end of the solenoid. This means that the field outside of the solenoid can be approximated to zero.

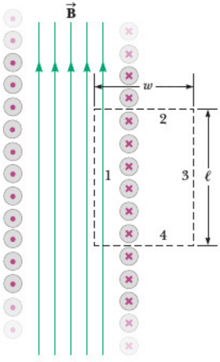
Magnetic Field Of A Solenoid

The magnetic field around a solenoid:



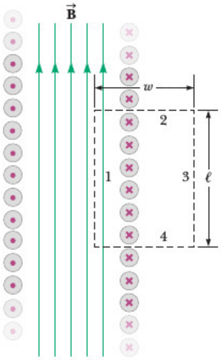
The solenoid acts as a magnet, with a north and south pole.

Applying Ampère's Law To A Solenoid



The field inside the solenoid is uniform and parallel to the axis of the solenoid, and approximately zero outside.
The closed path is taken to be a rectangle of length l and width w. This closed path is broken into four segments.

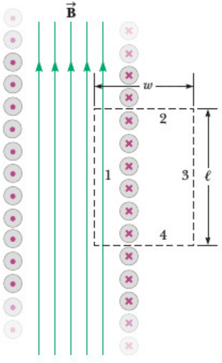
Applying Ampère's Law To A Solenoid



The sum of $B_{\parallel}\Delta l$ for each side of the rectangle gives

$$\sum B_{\parallel}\Delta l = B_{\parallel}\Delta l_1 + B_{\parallel}\Delta l_2 + B_{\parallel}\Delta l_3 + B_{\parallel}\Delta l_4$$

Applying Ampère's Law To A Solenoid



The only non-zero term is $B_{\|}\Delta l_1$, since the field is zero outside the solenoid (Δl_3) and perpendicular to the line segments Δl_2 and Δl_4.

Applying Ampère's Law To A Solenoid

The total current enclosed is the number of turns N within the closed path, multiplied by the current I, which passes through each turn.

$$\begin{aligned}\sum B_{\parallel}\Delta l &= B_{\parallel}\Delta l_1 = Bl \\ &= \mu_0(NI) \\ \therefore B &= \mu_0\frac{N}{l}I \\ &= \mu_0 nI\end{aligned}$$

where n is the number of turns per unit length.

Which Objects Are Magnets?

A current loop, such as a coil of wire with a current passing through it, can be shown to act as a magnet. A single coil of wire with a current in it has a north pole and a south pole.

If this is true for a current loop, should it not be true for any current moving a in closed loop?

What about the motion of electrons about the nucleus of an atom?

Why isn't every object magnetic?

It can be shown that the magnetic effect of one electron in an atom is mostly cancelled by another electron in the same atom moving in the opposite direction.

The overall magnetic effect of all the electrons within an atom is either zero or very small for most materials.

Which Objects Are Magnets?

Electrons in atoms not only "orbit" the nucleus, but also have a property called *spin*.

The spin property of an electron (or any other particle) is understood only using quantum mechanics. The electron can be thought of as a sphere that spins about an axis.

The "spinning" electron represents a charge in motion, thereby producing a magnetic field.

The magnetic effect due to the electron's spin is generally greater than the effect due to its orbital motion in the atom.

Electrons in an atom tend to pair up, each with opposite spins, therefore tend to cancel each other's magnetic effect.

Ferromagnetism

In some materials (e.g. iron, cobalt) the magnetic effect of the electron spins do not cancel completely.

Such materials are called *ferromagnetic*.

In ferromagnetic materials, large groups of atoms form domains that have spins aligned in one direction in that domain.

In general, the domains are all randomly oriented. In the presence of an external magnetic field, most of the domains tend to align themselves with the external field, resulting in a magnet.

Permanent magnets are ferromagnetic materials where the domains remain aligned even when the material is no longer in the presence of an external magnetic field.

Paramagnetism And Diamagnetism

Paramagnetic materials have magnetic moments (spins) that also align themselves in the presence of an external magnetic field. The magnetic effect is much weaker than in ferromagnetic materials. Aluminium and calcium are examples of paramagnetic materials.
In diamagnetic materials, the magnetic effect due to an external field is also weak (weaker than noth ferromagnetic and paramagnetic effects). The spins in a diamagnetic material tend to align themselves in a direction opposite to the external magnetic field.