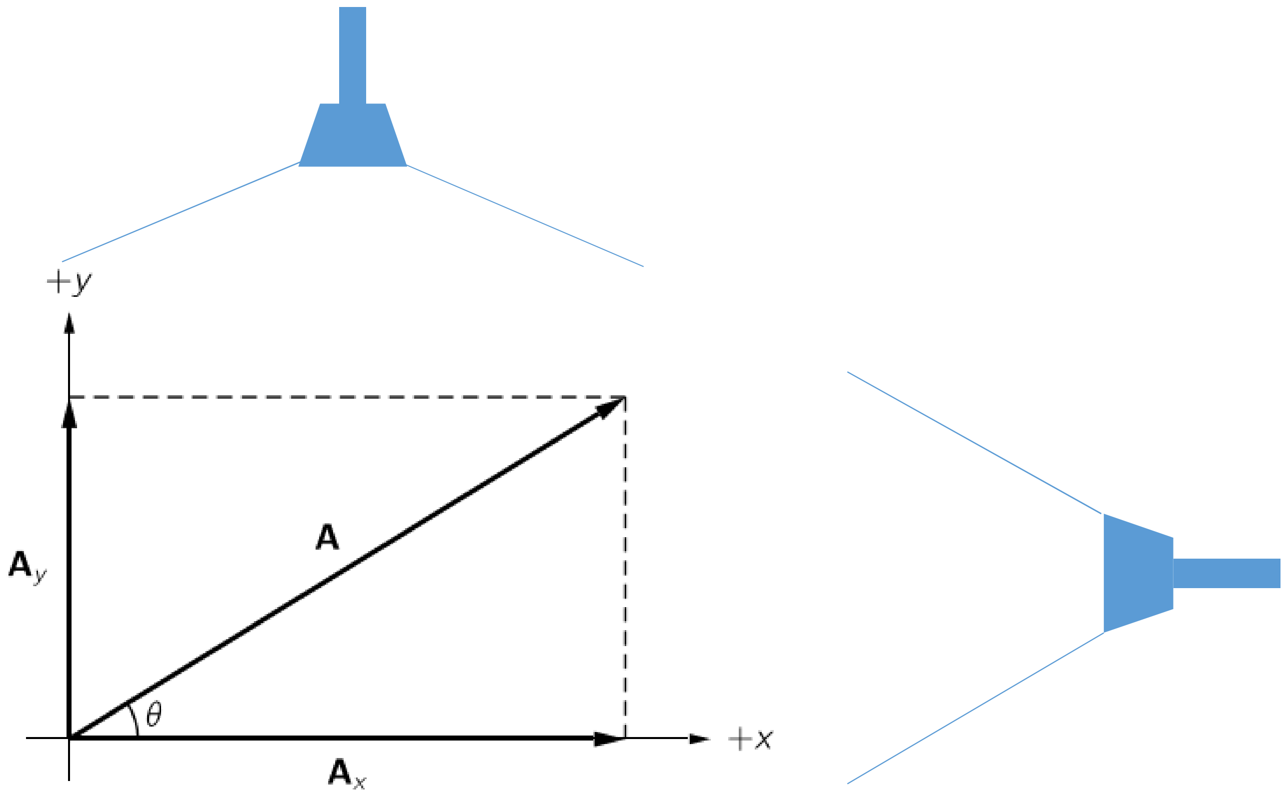
Components of a vector

Consider a vector (**A**), in the figure below, drawn such that its tail is at the origin of the Cartesian plane. The components of this vector are its projections along the x- and y-axis. *Informally, the components can be considered as the lengths of the shadows of the vector along the axes*. The x-component is denoted as $\mathbf{A}_x$ and the y-component is denoted as $\mathbf{A}_y$. From the figure it is evident that vector **A** is the sum or the resultant of its component. The components $\mathbf{A}_x$ and $\mathbf{A}_y$ are positive if they point towards the positive x- and y-axes, respectively. The magnitude of the components is denoted as A_x and A_y and they are called scalar components.



The resolution of vector

Knowing the magnitude and the direction of the vector we can determine its components using the trigonometric rations. This is known as **resolving vectors into its components**. In the figure below, if the magnitude of vector **A** is known and the direction is θ from the positive x-axis, the x- and y-components of vector **A** are:

$$A_x = A\cos\theta$$
and
$$A_y = A\sin\theta$$

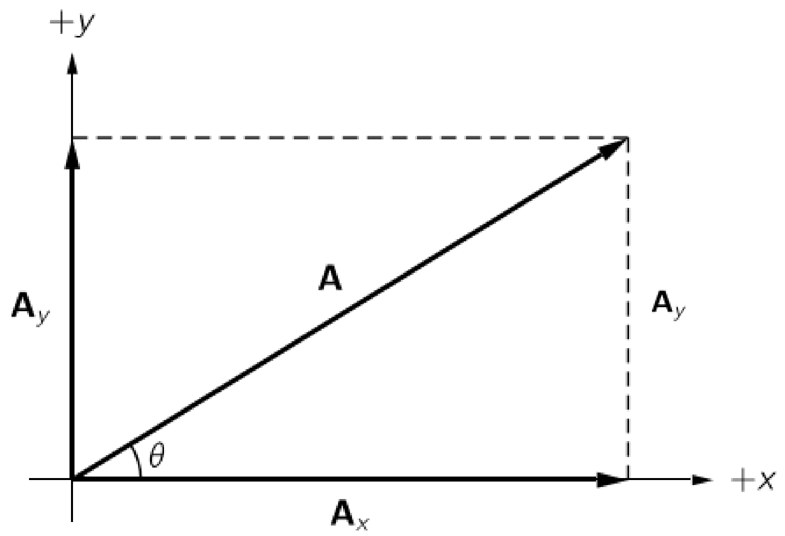
NB: The x- and y-components will always be given by $\mathbf{A_x = A\cos\theta}$ and $\mathbf{A_y = A\sin\theta}$ provided that **angle θ** is **between the vector and the x-axis**.

Conversely, if the components are known, the magnitude of the vector and its direction computed. The magnitude is given by Pythagoras theorem

$$A = \sqrt{A_x^2 + A_y^2}$$

And the direction is given by

$$\theta = tan^{-1}\left(\frac{A_y}{A_x}\right)$$

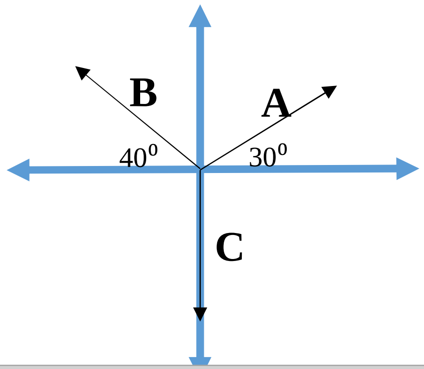


Addition of vectors using components

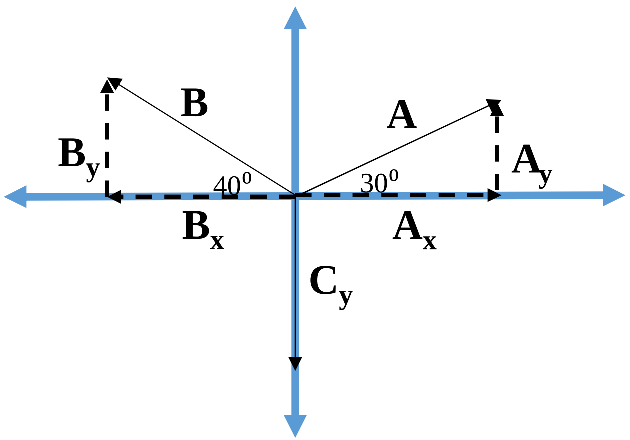
Consider three vectors **A**, **B** and **C.** Vector **A** forms an angle of 30^0 with the positive x-axis, vector **B** forms an angle of 40^0 with the negative x-axis and **C** points towards the negative y-axis. Find the resultant **R** of these vectors.

Method

1. Draw the vectors such that their tails are at the origin



2. Draw the components of vectors, indicate their directions

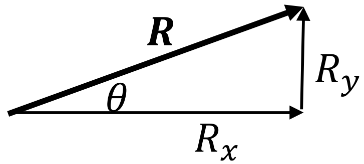


3. Computes the x-components of all the vectors and add them. Do the same on the y-components.

$$A_x = +A\cos 30° \qquad A_y = +A\sin 30°$$
$$B_x = -B\cos 40° \qquad B_y = +B\sin 40°$$
$$C_x = +C\cos 90° = 0 \qquad C_y = -C\sin 0° = -C$$

$$R_x = A_x + (-B_x) + C_x \qquad R_y = A_y + B_y + (-C_y)$$

4. Compute the magnitude of the resultant vector **R**



$$R^2 = R_x^2 + R_y^2 \qquad \mathrm{R} = \sqrt{R_x^2 + R_y^2}$$

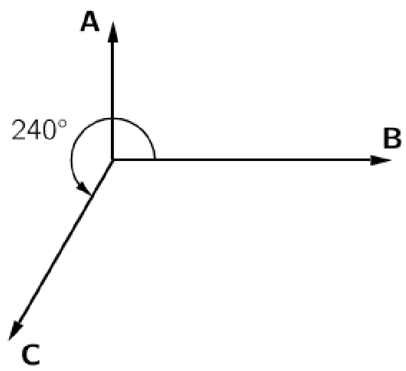
5. Find the direction of the resultant vector

$$\theta = tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Addition of vectors using components cont.

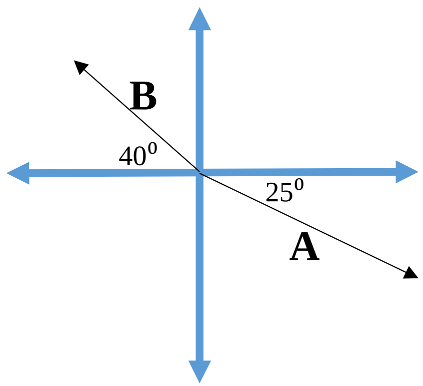
Example 1.7: Addition of vectors by adding components

Find the resultant of the three vectors in the drawing below. The vectors **A** and **B** are perpendicular to each other, and the magnitudes of **A**, **B** and **C** are 10, 20 and 15 units respectively.



Homework: Addition of vectors by adding components

Find the resultant of the two vectors in the drawing below. The vector **A** makes angle of 25^0 with the positive x-axis and vector **B** makes an angle of 40^0 with the negative x-axis. The magnitudes of **A** and **B** are 50 m, and 30 m units respectively.



R = 22.4 m
$\theta = 4.6°$