Physics 133: tutorial week 4
Ohm's law, electrical power, emf and internal resistance.

41. The heating element of a clothes drier has a resistance of $11\,\Omega$ and is connected across a $240\,\text{V}$ electrical outlet. What is the current in the heating element? $(22\,\text{A})$

$$V = IR \quad \therefore \quad I = \frac{240}{11} = 21.8\,\text{A}\,.$$

42. Three resistors, $25\,\Omega$, $45\,\Omega$ and $75\,\Omega$ are connected in series, and a $0.51\,\text{A}$ current passes through them. What is (a) the equivalent resistance and (b) the total potential difference across the three resistors? $(145\,\Omega;\quad 74\,\text{V})$

(a) $R_{\text{eq}} = R_1 + R_2 + R_3 = 25 + 45 + 75 = 145\,\Omega\,.$
(b) $V = IR = 0.51 \times 145 = 74\,\text{V}\,.$

43. The current in a series circuit is $15.0\,\text{A}$. When an additional $8.00\,\Omega$ resistor is inserted in series, the current drops to $12.0\,\text{A}$. What is the resistance in the original circuit? $(32\,\Omega)$

$$V = IR = 15.0 \times R = 12.0 \times (R + 8.0) \quad \text{which gives} \quad R = 32\,\Omega\,.$$

44. Suppose you want to run some apparatus that is $200\,\text{m}$ from the plug point. Each of the two wires connecting the apparatus to the $240\,\text{V}$ supply has a resistance per unit length of $0.006\,\Omega\,\text{m}^{-1}$. If the apparatus draws a current of $5.0\,\text{A}$, what will be the voltage drop across the lead, and what voltage will be applied to your apparatus? $(12\,\text{V};\quad 228\,\text{V})$

The total length of the wires to and from the apparatus is $200 \times 2 = 400\,\text{m}$. The total resistance of the lead is therefore $400 \times 0.006 = 2.4\,\Omega$. Using Ohm's law we find that the voltage drop across the lead $V = IR = 5.0 \times 2.4 = 12\,\text{V}$.

Since the lead is in series with the apparatus, the voltage drop across the apparatus must be The voltage of the supply minus the voltage drop across the leads. Thus $V = 240 - 12 = 228\,\text{V}$.

45. The element of an electric oven is designed to produce $3.0\,\text{kW}$ of heat when connected to a $220\,\text{V}$ source. What must be the resistance of the element? $(16.1\,\Omega)$

$$P = \frac{V^2}{R}, \quad \text{hence} \quad R = \frac{V^2}{P} = \frac{220^2}{3 \times 10^3} = 16.1\,\Omega\,.$$

46. A car starter motor draws $150\,\text{A}$ from the $12\,\text{V}$ battery. How much power is this? $(1.8\,\text{kW})$

$$P = VI = 12 \times 150 = 1800\,\text{W}\,.$$

47. How many kWh does a $1500\,\text{W}$ electric frying pan use in 15 minutes of operation? $(0.375\,\text{kWh})$

$$\text{Energy} = Pt = 1500 \times \frac{15}{60}\,\text{Wh} = 0.375\,\text{kWh}\,.$$

48. At 15 c per kWh, what does it cost to leave a 60 W light bulb on all day and night for one year? (R 78.84)

Power $= 60 \times 10^{-3} = 0.06\,\text{kW}$.
Time $= 365 \times 24 = 8760$ hours.
Energy $= Pt = 0.06 \times 8760 = 525.6\,\text{kWh}$.
Cost $=$ Energy $\times$ cost per unit $= 525.6 \times 15 = 7884$ cents $=$ R 78.84.

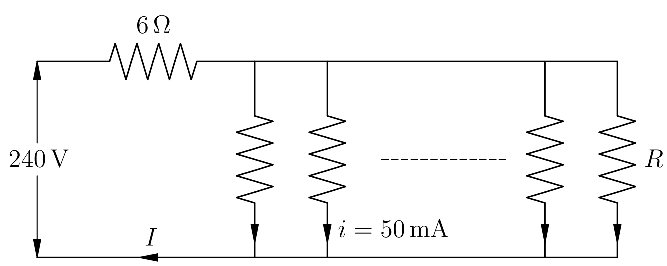
49. What is the total amount of energy stored in a 12 V, 60 Ah car battery when it is fully charged? (2592 kJ)

Energy $= V(It) = 12\,\text{V} \times 60\,\text{Ah} = 12 \times 60\,\text{VA} \times 60 \times 60\,\text{s} = 2.59 \times 10^3\,\text{kJ}$.

50. A person accidentally leaves a car with the lights on. The two front lights use 40 W each and the two rear lights use 6 W each. How long will a fresh 12 V battery last if it is rated at 75 Ah? Assume the full 12 V appears across each bulb. (9 hours and 47 minutes)

The total power consumed is $2 \times 40 + 2 \times 6 = 92\,\text{W}$.
The lights will therefore draw $I = P/V = 92/12 = 7.67\,\text{A}$.
If the time is t, then $7.67 \times t = 75\,\text{Ah}$, giving $t = 9.78$ hours $= 9$ hours 47 minutes.

51. Eight identical christmas-tree lights are connected in parallel to a 240 V source by two leads of total resistance $6.0\,\Omega$. If 50 mA flows through each bulb, what is the resistance of each, and what fraction of the total power is wasted in the leads? ($4752\,\Omega$; 1%)



There are 8 lamps each with resistance R connected in parallel. Thus

$$\frac{1}{R_{\text{lamps}}} = 8 \times \frac{1}{R}.$$

The resistance of the leads $(6.0\,\Omega)$ is in series with this resistance, hence

$$R_{\text{TOT}} = 6.0 + R_{\text{lamps}} = 6.0 + \tfrac{1}{8}R.$$

Also, the total current $I = 8 \times 50\,\text{mA} = 0.40\,\text{A}$. Using Ohm's law ($V = IR_{\text{TOT}}$), we find

$$R_{\text{TOT}} = 6.0 + \tfrac{1}{8}R = \frac{240}{0.40},$$

which gives $R = 4752\,\Omega$.

To determine the fraction of the power wasted in the leads, we need to calculate the total power used by the lamps plus the leads and the power dissipated through the leads alone.

$P_{\text{TOT}} = VI = 240 \times 0.40 = 96\,\text{W}$.
$P_{\text{leads}} = I^2R = 0.40^2 \times 6.0 = 0.96\,\text{W}$.

Hence, the % of the power wasted in the leads $= \dfrac{0.96}{96} \times 100 = 1\%$.

52. A $2.00\,\Omega$ resistor is connected across a $6.00\,\text{V}$ battery. The voltage between the terminals of the battery is observed to be only $4.90\,\text{V}$. Find the internal resistance of the battery. $(0.45\,\Omega)$

The current through the resistor is

$$I = \frac{V}{R} = \frac{4.90}{2.00} = 2.45\,\Omega\,.$$

Using $\mathscr{E} = V + Ir$, we find

$$r = \frac{\mathscr{E} - V}{I} = \frac{6.00 - 4.90}{2.45} = 0.45\,\Omega\,.$$

53. A battery produces $50.0\,\text{V}$ when $5.0\,\text{A}$ is drawn from it and $48.5\,\text{V}$ when $20.0\,\text{A}$ is drawn. Calculate the emf and internal resistance of the battery. $(50.5\,\text{V}; \quad 0.1\,\Omega)$

$\mathscr{E} = V + Ir$, hence $\mathscr{E} = 50 + 5.0r$ and $\mathscr{E} = 48.5 + 20.0r$.
Solving these equations for $\mathscr{E}$ and r we obtain $\mathscr{E} = 50.5\,\text{V}$ and $r = 0.1\,\Omega$.

54. A battery whose emf is $6.0\,\text{V}$ and internal resistance is $1.0\,\Omega$ is connected to a circuit whose net resistance is $23\,\Omega$. What is the terminal voltage of the battery? $(5.75\,\text{V})$

The terminal voltage equals the voltage drop across the circuit. Thus $V = IR = \mathscr{E} - Ir$, which gives

$$I = \frac{\mathscr{E}}{R + r} = \frac{60}{23 + 1} = 0.25\,\text{A}\,.$$

The terminal voltage of the battery is therefore $V = IR = 0.25 \times 23 = 5.75\,\Omega$.

55. A dry cell having an emf of $1.55\,\text{V}$ and an internal resistance of $0.08\,\Omega$ supplies current to a $2.0\,\Omega$ resistor.

(a) Determine the current in the circuit.

(b) Calculate the terminal voltage of the cell. $(0.745; \quad 1.49\,\text{V})$

(a) $\mathscr{E} = V + Ir = IR + Ir$, hence $I = \dfrac{\mathscr{E}}{R + r} = \dfrac{1.55}{0.08 + 2.0} = 0.745\,\text{A}$.

(b) The terminal voltage equals the voltage drop across the $2.0\,\Omega$ resistor, hence
$V = IR = 0.745 \times 2.0 = 1.49\,\text{V}$.

56. How many cells, each having an emf of $1.5\,\text{V}$ and an internal resistance of $0.50\,\Omega$ must be connected in series to supply a current of $1.0\,\text{A}$ to operate an instrument having a resistance of $12\,\Omega$? (12)

For n cells connected in series, $\mathscr{E}_{\text{TOT}} = \sum \mathscr{E}_i = n\mathscr{E}$ and $r_{\text{TOT}} = \sum r_i = nr$. Here $\mathscr{E}_i$ and r_i are the emf and internal resistances of the individual cells. Thus

$\mathscr{E}_{\text{TOT}} = IR + Ir_{\text{TOT}}$ yields

$n \times 1.5 = 1.0 \times (12 + n \times 0.50)$ and hence $n = 12$.

57. A battery has an internal resistance of $0.50\,\Omega$. A number of identical light bulbs, each with a resistance of $15\,\Omega$, are connected in parallel across the battery terminals. The terminal voltage of the battery is observed to be half the emf of the battery. How many bulbs are connected? (30)

Suppose there are n lightbulbs. The total resistance of all the lightbulbs in parallel may therefore be determined from

$$\frac{1}{R} = \frac{n}{15} \quad \text{or} \quad R = \frac{15}{n}.$$

The terminal voltage across the battery equals the voltage drop across the lightbulbs. But $V = \frac{1}{2}\mathscr{E}$, hence

$$V = \tfrac{1}{2}\mathscr{E} = IR = \frac{15I}{n}.$$

Also

$$\mathscr{E} = V + Ir, \quad \text{hence} \quad 2 \times IR = IR + Ir \quad \text{or} \quad R = r.$$

Using $R = 15/n$ and $r = 0.50\,\Omega$, we find $n = 30$.

58. The internal resistance of a $1.35\,\text{V}$ mercury cell is $0.04\,\Omega$, whereas that of a $1.5\,\text{V}$ dry cell is $0.50\,\Omega$. Explain why three mercury cells can more effectively power a $2\,\text{W}$ hearing aid that requires $4.0\,\text{V}$ than can three dry cells. Include relevant calculations in your answer.

We can obtain the current drawn by the hearing aid from $P = VI$, where $P = 2\,\text{W}$ and $V = 4.0\,\text{V}$, which gives $I = 0.50\,\text{A}$. The resistance of the hearing aid is $R = \frac{V}{I} = \frac{4.0}{0.50} = 8.0\,\Omega$.

For the mercury cells: $\mathscr{E}_\text{m} = 3 \times 1.35 = 4.05\,\text{V}$ and $r_\text{m} = 3 \times 0.04 = 0.12\,\Omega$.
For the dry cells: $\mathscr{E}_\text{d} = 3 \times 1.5 = 4.5\,\text{V}$ and $r_\text{d} = 3 \times 0.5 = 1.5\,\Omega$.

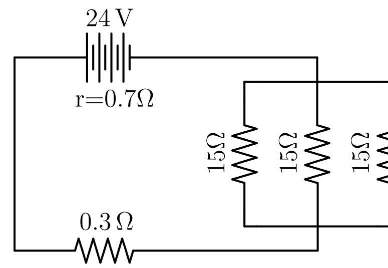
We can now find the current that will flow in the hearing aid from $\mathscr{E} = IR + r$ for each battery. Thus

For the mercury cells: $$i = \frac{\mathscr{E}_\text{m}}{r + R} = \frac{4.05}{0.12 + 8} = 0.499\,\text{A}.$$

For the dry cells: $$i = \frac{\mathscr{E}_\text{d}}{r + R} = \frac{4.5}{1.5 + 8} = 0.474\,\text{A}.$$

Clearly the mercury cells provide a current closer to the desired value of $0.5\,\text{A}$.

59. A battery of emf 24 V and internal resistance 0.7 Ω is connected to three 15 Ω coils arranged in parallel, and a 0.3 Ω resistor is connected in series as shown in the diagram. Determine

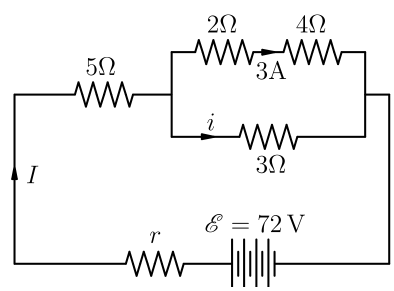


(a) the current in the circuit,
(b) the current in each parallel branch,
(c) the potential difference across the parallel group and across the 0.3 Ω resistance,
(d) the terminal voltage of the battery while it delivers current.

(4 A; 1.33 A; 20 V; 1.2 V; 21.2 V)

(a) $R_{\text{eq}} = 0.3 + \dfrac{15}{3} = 5.3\,\Omega$.
Using $\mathscr{E} = 24\,\text{V}$, $r = 0.7\,\Omega$ and $R = 5.3\,\Omega$ in $\mathscr{E} = I(R + r)$, we find $I = 4\,\text{A}$.
(b) Since the resistors in the parallel circuit are all the same, the current $I = 4\,\text{A}$ divides equally through each resistor, hence $i = \frac{1}{3} \times 4 = 1.33\,\text{A}$.
(c) $V_{\|} = IR_{\|} = 4 \times \dfrac{15}{3} = 20\,\text{V}$.
$V_{0.3} = IR_{0.3} = 4 \times 0.3 = 1.2\,\text{V}$.
(d) The terminal voltage equals the voltage drop across the external circuit, hence
$V = V_{\|} + V_{0.3} = 21.2\,\text{V}$.

60. The current in the 2 Ω resistor in the circuit opposite is 3 A. Determine



(a) the current i in the 3 Ω resistor
(b) the total current I
(c) the terminal p.d. of the battery
(d) the internal resistance r of the battery.

(6 A; 9 A; 63 V; 1 Ω)

(a) The voltage drop across the 3 Ω resistor is equal to the drop across the (2 + 4) Ω resistors. Hence $3 \times 6 = i \times 3$ which gives $i = 6\,\text{A}$.
(b) $I = 3 + 6 = 9\,\text{A}$.
(c) $R_{\text{eq}} = 5 + \dfrac{6 \times 3}{6 + 3} = 7\,\Omega$.
The terminal voltage is therefore equal to the voltage drop across this equivalent resistor. Hence
$V = IR_{\text{eq}} = 9 \times 7 = 63\,\text{V}$.
(d) $\mathscr{E} = V + Ir$ gives $r = \dfrac{\mathscr{E} - V}{I} = \dfrac{72 - 63}{9} = 1\,\Omega$.